Quantitative Macroeconomics

Hopenhayn & Rogerson (1993)

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UnB

Introduction

Hopenhayn & Rogerson (1993): Quantitative application of the industry dynamics model.

- Large volume of job creation and destruction at the firm level that does not show up in the aggregate.
- Changes in employment at the firm level tend to be lumpy.
- How to consider these facts? What are the consequences of policies that make it costly to fire workers?
- Introduce adjustment costs $\Rightarrow$ induce misallocation of resources across heterogeneous producers.
- Also, introduce general equilibrium to the household side.

Employment Reallocation across Firms (U.S)

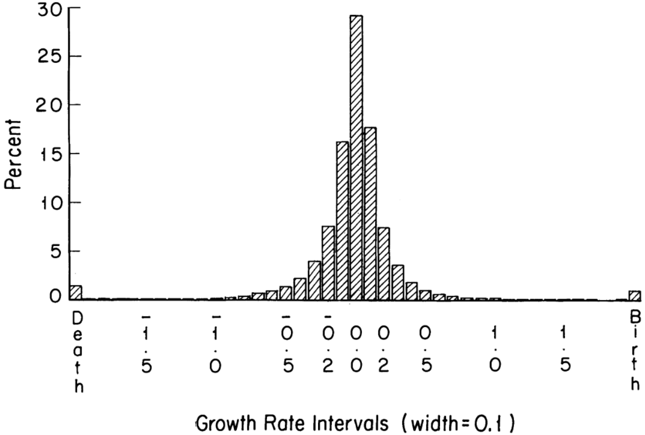


FIGURE Ib
Size-Weighted Growth Rate Distribution

Model

- Focus on stationary equilibrium.
- Individual firm productivities, z, follow a first-order Markov process with distribution function $F(z'|z)$.
- Entrants draw their initial productivity from a fixed distribution $z_0 \sim G(z)$.
- Firms face convex labor adjustment costs, fixed cost and entry cost.
- Households supply labor elastically.

Household

- The representative household solves the following problem:

$$\max_{C_t, N_t} \sum_{t=0}^{\infty} \beta^t \ln C_t + AN_t \qquad \text{s.t.} \qquad p_t C_t = w_t N_t + \Pi_t + T_t,$$

  where:

  - N_t: Household's labor supply.
  - Π_t: Firm's profit.
  - T_t: Transfers from government.

- The linear labor supply decision comes from **Rogerson's (1988) Employment Lotteries Trick**.
- Note that the problem can be solved as a sequence of static problems.
- We solve for the steady-state so safely ignore time subscripts. Normalize $w = 1$

Household

- The problem:

$$\max_{C,N} \ln C + AN \qquad \text{s.t.} \qquad pC = N + \Pi + T,$$

  gives the household demand for the final good and the labor supply decision:

$$C = \frac{1}{Ap} \qquad \text{and} \qquad N = A - \Pi - T.$$

- Write them in the general form: $C = C^h(p, \Pi + T)$ and $N = N^s(p, \Pi + T)$.

Firms

- Firms produce the final good with: $y = zf(n)$, where $f(n)$ is a DRS technology.
- They face an adjustment cost function representing **firing costs**:

$$g(n_t, n_{t-1}) = \tau \max\{0, n_{t-1} - n_t\}, \qquad \tau \geq 0.$$

- The (static) profit problem is:

$$pzf(n_t) - n_t - g(n_t, n_{t-1}) - pc_f$$

- **Key**: Past employment n_{t-1} is a state variable.

Timing Within a Period

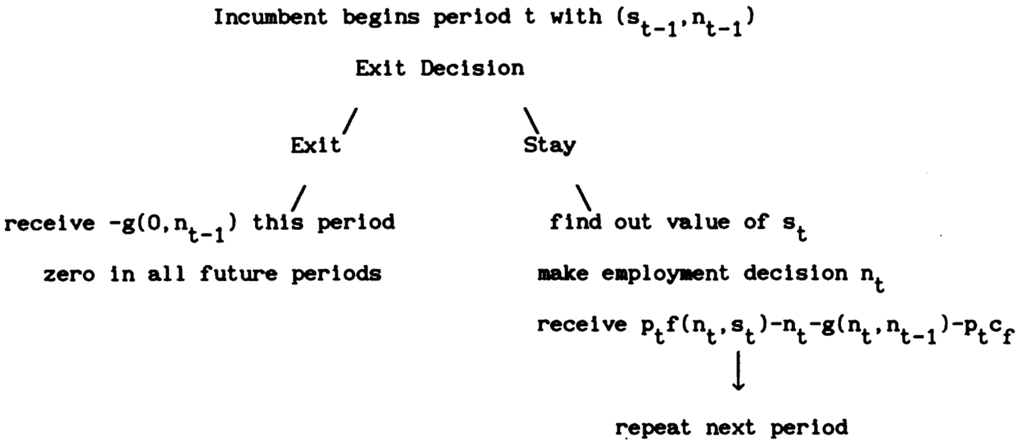


FIG. 1.—Timing of decisions

Incumbent Firms

- The value function of incumbent is:

$$V(z,n) = \max_{n' \geq 0} \left\{ pzf(n') - n' - g(n',n) - pc_f + \beta \max \left[-g(0,n'), \int V(z',n') dF(z'|z) \right] \right\}$$

  and the policy functions are $n' = n^d(z,n;p)$ and $\chi(z,n;p) \in \{0,1\}$.

- Firms that exit have to pay the firing cost of their labor force and then receive zero in the following periods.

Entrants and Free Entry Condition

- Potential entrants are ex-ante identical.

- An entrant firm must pay the entry cost $c_e > 0$ to set-up the plant and draw $z \sim G(z)$. Start producing next period with $n_{t-1} = 0$.
  - ps. in the original paper, H&R assume that entrants produce in the same period.

- There is a $M \geq 0$ mass of entrants. In equilibrium, the **free entry condition** reads:

$$\beta \int V(z, 0; p) dG(z) \leq c_e.$$

with strict equality if $M > 0$.

Stationary Distribution

- Let $\mu(z,n)$ denote the distribution of firms across the state. The distribution follows the law of motion

$$\mu_{t+1}(z',n') = \int Q(z',n'|z,n)d\mu_t(z,n) + M_{t+1}G(z')\mathbf{1}_{[n'=0]}.$$

where the transition function is given by the labor and exit policy function:

$$Q(z',n'|z,n) = F(z'|z)(1-\chi(z,n))\mathbf{1}_{[n'=n^d(z,n)]}.$$

- In the **stationary equilibrium**, we have $\mu_{t+1} = \mu_t = \mu$.

- The stationary distribution depends on two equilibrium objects: $\mu(p,M)$. Again, linearity implies that $\mu(p,M) = M \times \mu(p,1)$.

Aggregation

- Aggregate production and labor demand:

$$Y(p,M) = \int (zf(n^d(z,n;p)) - c_f)d\mu \qquad \text{and} \qquad N^d(p,M) = \int n^d(z,n;p)d\mu + Mc_e$$

- Expected firing tax revenue for a firm with state (z,n) is:

$$r(z,n;p) = [1-\chi(z,n)]\mathbb{E}_{z'|z}[g(n^d(z',n^d(z,n)),n^d(z,n))] + \chi(z,n)g(0,n')$$

and aggregate tax revenue $T(p,M) = \int r(z,n;p)d\mu$.

- Aggregate profits:

$$\Pi(p,M) = pY - N^d - T = \int \pi(z,n;p)d\mu - Mc_e.$$

Equilibrium

- We can solve for the equilibrium using the same approach as in the original Hopenhayn model.
- **Step 1**: Guess a price, p^*, and solve for the dynamic programming problem of the incumbents.
- **Step 2**: Check if p^* satisfies the free entry: $\beta \int V(z, 0; p) dG(z) = c_e$. If no, return to step 1.
- Note that the dynamic programming problem is more evolved than Hopenhayn since we must find the labor decision as well.

Equilibrium

- **Step 3**: Given p^*, and the policy functions, assume $M = 1$ and solve for the stationary distribution $\mu(p^*, 1)$.
  - Again, because labor is a state variable, solving for the invariant distribution is harder. One option is to use non-stochastic simulation.
- **Step 4**: Use either the goods market or the labor market clearing condition to solve for M.
  - The functional forms used in the household problem make solving for the goods market easier:

$$Y(p^*, M) = M \int (zf(n^d(z, n; p^*)) - c_f) d\mu(p^*, 1) = C(p^*)$$

Firing Taxes

- If there are no adjustment costs, $(\tau = 0)$, the marginal product of labor equalize across firms

$$zf'(n') = \frac{1}{p},$$

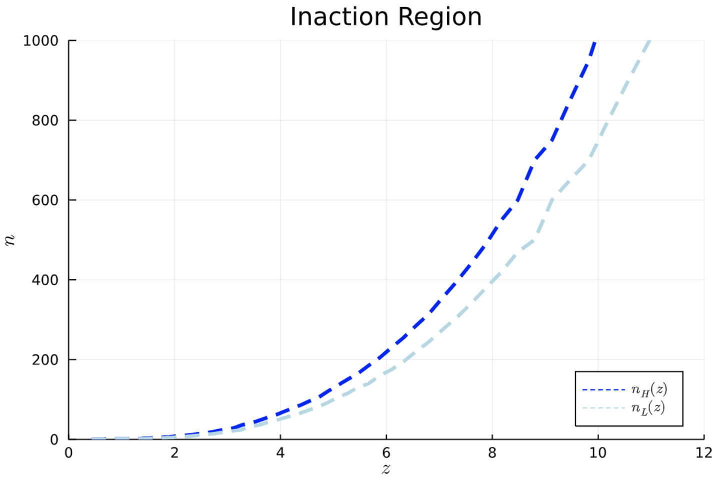
  and we can easily see that $n^d(z, n)$ is independent of the previous employment n.

- When there are adjustment costs, $(\tau > 0)$, the firm may not find optimal to re-adjust labor - even if z has changed.

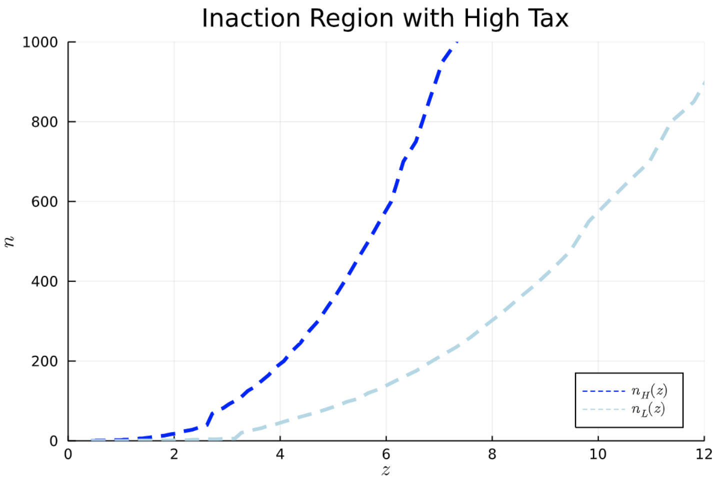
- Hence, there is an **inaction region**:

$$n^d(z, n) = n' = n, \qquad \text{if} \qquad n \in (n_L(z), n_H(z))$$

Firing Taxes



Firing Taxes (Tax $10\times$)



Higher τ increases the inaction region.

Firing Taxes

- The adjustment cost implies that adjustment is **lumpy** (if the adjustment cost is not quadratic).
  - If the adjustment cost is quadratic, firms will adjust slowly (no inaction region).
- In H&R the linear adj. cost induces the inaction region.
- Nowadays, it is more common a combination of fixed + symmetric quadratic adjustment.
  - Nice property of having the inaction region, + analytical properties of quadratic adjustment.
- Adjustment costs are less important if shocks are **very persistent**:
  - High persistent shocks $\Rightarrow$ efficient scale does not change often.
  - Low persistent shocks $\Rightarrow$ efficient scale changes often.

Firing Taxes

- Because firms do not adjust their labor, the MPL does NOT equalize across producers $\Rightarrow$ increase misallocation in the economy!
  - Misallocation (in %) for firm i: $\frac{|MPN_i - 1/p|}{1/p} \times 100$.

- Firing cost reduces labor reallocation:
  - Low-productivity firms should be shrinking;
  - High-productivity firms should be expanding;

- The tax also prevents inefficient firms from exiting.

- Note that misallocation here is induced by an aggregate friction. In more sophisticated models, the misallocation can be **firm-specific**.

Numerical Simulation: $\tau = 0.1$

```
Model Stats

Price: 1.4437674638040225
Avg. Firm Size: 111.09469333242433
Exit/entry Rate: 0.2732168521382527
Avg. Productivity: 4.323205664819014
Avg. Misallocation (%): 4.692349861632813
Agg. Output: 69.2632314462338
Agg. Labor Supply: 83.54227263629872
Agg. Tax Revenue: 1.7626005743152082
Agg. Profits: 14.695126789386073
Mass of Firms: 0.7167378826624267
Mass of Entrants: 0.19582486810926455

Size       10    20    50    100   1000
Firm Share 0.118 0.197 0.433 0.675 0.994
Emp. Share 0.005 0.016 0.094 0.252 0.915
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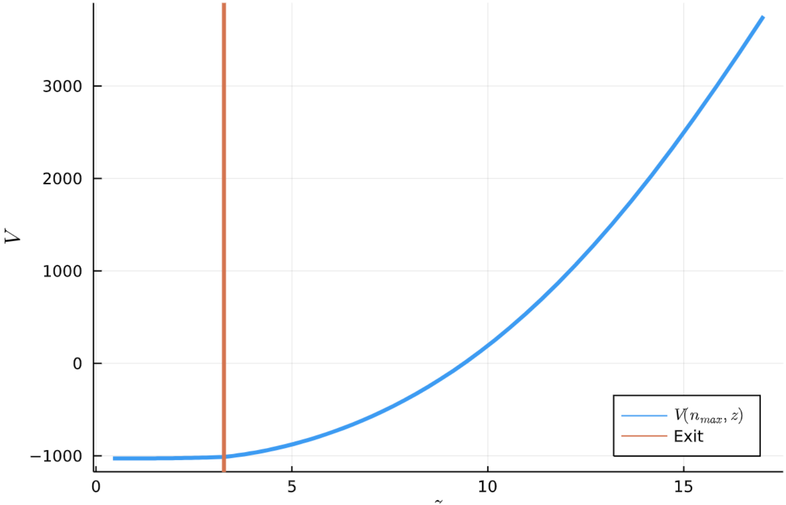
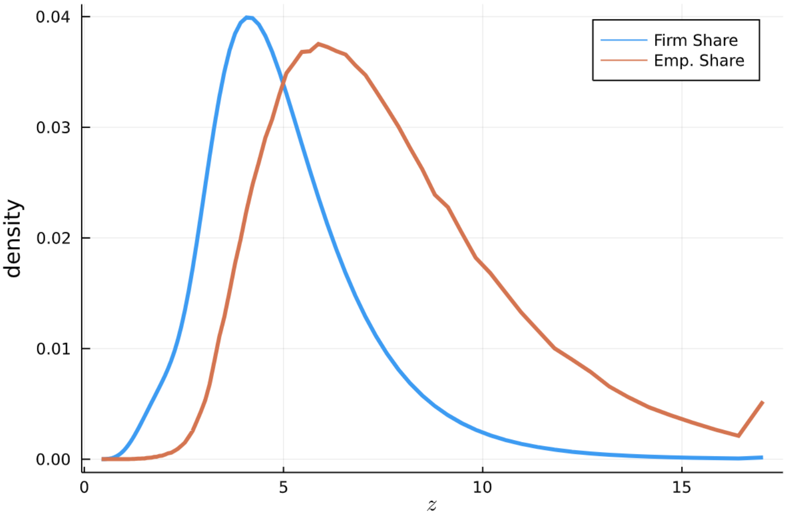
Numerical Simulation: $\tau = 0.1$



Numerical Simulation: $\tau = 0.1$



Numerical Simulation: $\tau = 0$

```
Model Stats

Price: 1.415691805169249
Avg. Firm Size: 110.28465494925103
Exit/entry Rate: 0.29482477569028853
Avg. Productivity: 4.33671698867755
Avg. Misallocation (%): 0.6433331033616971
Agg. Output: 70.6368431567242
Agg. Labor Supply: 84.65635014651724
Agg. Tax Revenue: 0.0
Agg. Profits: 15.34364985348276
Mass of Firms: 0.7286582187308964
Mass of Entrants: 0.21482649589222172

Size        10    20    50    100   1000
Firm Share 0.109 0.195 0.472 0.700 0.994
Emp. Share 0.005 0.017 0.107 0.262 0.905
```

No Taxes: Prices are lower, output is higher, profits are higher, more entry/exit, and less misallocation.

Numerical Simulation: High Tax ($\tau = 1.0$)

```
Model Stats

Price: 1.5654051097501829
Avg. Firm Size: 129.41346103152415
Exit/entry Rate: 0.1968934068638075
Avg. Productivity: 4.197329231590462
Avg. Misallocation (%): 24.061028775528854
Agg. Output: 63.881227534742514
Agg. Labor Supply: 84.86324401222404
Agg. Tax Revenue: 7.5172181798792455
Agg. Profits: 7.619537807896725
Mass of Firms: 0.6363884375295689
Mass of Entrants: 0.12530068755393214

Size       10    20    50    100   1000
Firm Share 0.126 0.141 0.268 0.576 0.995
Emp. Share 0.002 0.005 0.040 0.226 0.948
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Conclusion

- H&R $\Rightarrow$ application of the Hopenhayn firms' dynamics model.
- Attempts to match the facts on job reallocation across firms.
- Study the effect of a firing cost.
- The friction induces misallocation of resources $\Rightarrow$ reduces aggregate productivity.