Quantitative Macroeconomics
Problem Set 2

1. **(The Hopenhayn & Rogerson Model)**. Consider the Hopenhayn & Rogerson model. Every period the representative household decides how much to consume and how much labor to supply to the market. The problem is summarized as:

$$\max_{C,N} \ln C - AN \quad \text{s.t.} \quad pc = N + \Pi + T. \tag{1}$$

The solution is given by: $C = 1/Ap$ and $N = 1/A - \Pi - T$, where Π is the aggregate profits and T is the aggregate tax revenue.

Incumbent firms produce the final good y according to the following production function: $y = zn^\alpha$, where n denotes the labor hired by the firm. They are subject to a per-period fixed cost c_f and a firing tax $g(n_t, n_{t-1}) = \tau \max\{0, n_{t-1} - n_t\}$. The productivity, z, follows an AR(1):

$$\log z_t = \mu(1-\rho) + \rho \log z_{t-1} + \sigma\varepsilon_t \qquad \varepsilon \sim N(0,1).$$

At the beginning of the period, before the realization of z, incumbents decide whether to continue operations or exit. In case the firm decides to exit, it receives $-g(0, n_{t-1})$ this period and 0 in all the others. The value function of incumbent reads:

$$V(z,n;p) = \max_{n'}\{pz(n')^\alpha - n' - pc_f - g(n',n) + \beta \max[-g(0,n'), \mathbb{E}_{z'|z}V(z',n';p)]\},$$

where p is the price of the final good, and the wage, w, is normalized to one. Denote the firm's policy function as $n' = n^d(z,n)$ and $\chi = \{0,1\}$, where $\chi = 1$ denotes the exit decision.

Potential entrants are ex-ante identical. They must pay an entry cost, c_e, and start producing in the next period. They draw their productivity from the distribution $G(z)$. Let $M \geq 0$ be the mass of entrants, the free entry condition implies that in equilibrium:

$$V^e(p) = \beta \int V(z,0;p)dG(z) - c_e \leq 0 \tag{2}$$

with equality whenever the mass of entrants is positive, $M > 0$.

We solve for the stationary equilibrium, the equilibrium where prices are constant and there is an invariant distribution $\mu(z,n)$ over the employment-productivity space. The market clearing conditions of the goods and labor market are:

$$Y = \int z(n'(z,n))^\alpha - c_f d\mu(z,n) = C \tag{3}$$

$$N = L^d + Mc_e. \tag{4}$$

(a) Outline a computational algorithm to find a stationary competitive equilibrium of the model.

(b) Solve the model in the computer using the following parameters: $\alpha = 2/3$, $\beta = 0.8$, $A = 0.01$, $c_e = 20.0$, $c_f = 20.0$, $\rho = 0.9$, $\sigma = 0.2$ and $\mu = 1.0$. Discretize the productivity process using the Tauchen algorithm. Make sure to have enough nodes (i.e., $n_z \geq 45$).

The employment grid is discretized between $n_{n,1} = 0$ and $n_{n,max} = 10000$. An easy way to discretize the employment space is to assume a vector of the type:

$$g_N = [0(1)20; 22(2)100; 105(5)500; 550(50)1000; 1100(100)5000; 5500(500)10000]',$$

where the number inside the () denotes the step size between grid points.[1]

Solve the model for $\tau = 0.1$ and show: (i) the average firm size; (ii) the average firm productivity; (iii) exit/entry rate; and (iv) the average misallocation of the economy, where misallocation of firm i is defined as

$$\text{misalloc}_i = \frac{|MPN_i - 1/p|}{1/p},$$

and MPN_i is the marginal product of labor.

(c) Solve the model again for no tax $\tau = 0.0$ and a high firing tax $\tau = 0.5$ how misallocation has changed in the economy?

(d) Solve the model for a low persistence productivity process $\rho = 0.5$. Compute the statistics for the three cases: $\tau = (0.0; 0.1; 0.5)$. How does the persistence of the stochastic process matter for the impact of the firing tax?

2. **(Labor Tax in the Hopenhayn Model)**. Consider the standard Horpenhayn & Rogerson model presented in the previous question. The parameter's values are the same as the previous question.

Suppose there is a government deciding between two types of taxes: a firing tax as modeled in the previous question, or a traditional labor tax of τ_n of the equilibrium wage. The value function of the incumbents read:

$$V(z, n; p) = \max_{n'}\{pz(n')^\alpha - n'(1 + \tau_n) - pc_f - g(n', n) + \beta \max[-g(0, n'), \mathbb{E}_{z'|z}V(z', n'; p)]\}.$$

The government already has a firing tax in place and it is thinking about whether to move to the other tax scheme. The reform should be revenue-neutral, i.e, it should yield the same tax revenue as before.

(a) Solve the model without the labor tax, $\tau_n = 0.0$, but with a firing tax of $\tau = 0.1$. Report the aggregate tax revenue, the aggregate output, average misallocation, average productivity, and the equilibrium price.

[1] For instance, $[0(1)3; 4(2)10] = [0, 1, 2, 3, 4, 6, 8, 10]$.

(b) Shut down the firing tax $\tau = 0.0$. Find the labor tax $\tau_n > 0$ that yields the same aggregate tax revenue (if it exists).[2] Report the aggregate output, average misallocation, average productivity, and the equilibrium price. What is the tax scheme that maximizes aggregate output?

[2]You can find it by trying different taxes rates or can write a root-finding function. Note that an exact tax rate may be hard to find because of the grid approximation. In this case, feel free to report an approximation.