Quantitative Macroeconomics

Fiscal Policy in HANK: The Intertemporal Keynesian Cross (Auclert, Rognlie and Straub, 2018, WP)

Tomás R. Martinez

UnB

References

- Auclert, Rognlie and Straub (2018, NBER WP)*. The Intertemporal Keynesian Cross.
- Also check their NBER summer course notes here.
- Complementary reference: Hagedorn, Manovskii, and Mitman (2019, WP).

Introduction

- What is the effect of an increase in **government spending**?
  - Does modeling HA-agents matter?
  - Should the fiscal policy be **deficit-financed** or should the government balance its budget all periods?
- New set of moments are key for the results $\Rightarrow$ **Intertemporal Marginal Propensities to Consume** (iMPCs).
  - What the data of iMPCs look like?
  - What kind of models match the data?
    - Heterogeneous Agents (HA), Two Agents (TA), Representative Agent (RA)?

A General Model

- Unit mass of individuals that live for $t = 1, .., \infty$.
- There is NO aggregate uncertainty, but agents may be subject to idiosyncratic shocks.
  - Idiosyncratic ability state e follows a Markov process with transition matrix Π.
  - Stationary distribution of state e is $\pi(e)$, average ability is normalized to one, i.e., $\sum_e \pi(e)e = 1$.
- Asset markets may or may not be complete, and There could be many assets with different liquidity.
- Governments may carry debt but must satisfy its intertemporal budget constraint.
- Flexible prices, but wage rigidity.
- **Simplifications:** no investment/capital, passive monetary policy.

Household Problem

- Household i enjoys consumption and gets disutility from labor:

$$
\begin{aligned}
\max \quad & \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ u(c_{it}) - v(n_{it}) \right\} \\
\text{s.t.} \quad & c_{it} + \sum_j a_{it}^j = z_{it} + (1 + r_{t-1}) \sum_j a_{it-1}^j \\
& a_{it}^j \in \mathcal{A}_{it}^j
\end{aligned}
$$

where z_{it} is the after-tax income and can capture progressive taxation:

$$
z_{it} \equiv \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it} \right)^{1-\lambda}
$$

- Note that the structure allows different assets j and a general asset-market structure, $\mathcal{A}_{it}^j$ (incomplete markets, different liquidity, etc).

Wage Rigidity

- Prices are flexible, but wages are sticky (see Erceg et al (2000) or Galí's book Chapt. 6). Introduce rigidity in layers so all HH work same number of hours $n_{it} = N_t$.

- There is a continuum of symmetric unions $k \in [0, 1]$.
  - Every worker i sells n_{ikt} hours to union k.
  - Each union aggregates efficient units of work into a union-specific task: $N_{kt} = \int e_{it} n_{ikt} di$.

- A competitive labor packer then package these tasks into aggregate employment using the CES:

$$N_t = \left(\int_k N_{kt}^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$$

  - The packer sells N_t to the aggregate firm that produces the final good.

Wage Rigidity: Packers

- The labor packer's demand tasks from the unions. The problem:

$$\max_{N_{kt}} \quad W_t N_t - \int W_{kt} N_{kt} dk \qquad \text{s.t.} \quad N_t = \left(\int_k N_{kt}^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$$

- Solution implies the following demand for union tasks and wage index:

$$N_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} N_t, \qquad \text{and} \qquad W_t = \left(\int W_{kt}^{1-\epsilon} dk \right)^{1/(1-\epsilon)}.$$

Wage Rigidity: Unions

- Unions set wages W_{kt} taking as given demand for their tasks N_{kt}.
- Workers do not like wage adjustments, so unions decide the wages to maximize discounted average utility of the workers subject to adjustment costs:

$$\max_{\{W_{kt+\tau}\}} \sum_{\tau \geq 0} \beta^{t+\tau} \left(\int \{u(c_{it+\tau}) - v(n_{it+\tau})\} d\Psi_{it+\tau} - \frac{\psi}{2} \left(\frac{W_{kt+\tau}}{W_{kt+\tau-1}} - 1 \right)^2 \right)$$

subject to

$$N_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} N_t \qquad \text{and HH budget constraint.}$$

New Keynesian Phillips Curve

- After some boring derivations here, since unions are symmetric, we can show:
  - All unions set the same wage, $W_{kt} = W_t$;
  - All HH work the same number of hours;

- It implies a non-linear **New Keynesian (Wage) Phillips Curve**:

$$\pi_t^w(1+\pi_t^w) = \frac{\epsilon}{\psi}\int N_t \left\{ v'(n_{it}) - \frac{(\epsilon-1)}{\epsilon}\frac{\partial z_{it}}{\partial n_{it}} u'(c_{it}) \right\} d\Psi_{it} + \beta \pi_{t+1}^w(1+\pi_{t+1}^w)$$

  - Conditional on future wage inflation, unions set higher nominal wages when MRS between n_{it} and c_{it} exceeds a marked-down average of mg. after-tax income from extra hours.
  - In the absence of rigidity: $v'(n_{it}) = \frac{(\epsilon-1)}{\epsilon}\frac{\partial z_{it}}{\partial n_{it}} u'(c_{it})$

Production Function

- Let X_t be the TFP. Assume no capital and CRS, aggregate production is given by:

$$Y_t = X_t N_t$$

- Due to perfect competition and flexible prices, the final goods price is given by:

$$P_t = \frac{W_t}{X_t} \quad \Rightarrow \quad \frac{W_t}{P_t} = X_t.$$

- Assume $X_{ss} = 1$, so in absence of TFP shocks, real wage is equal to one.
- Goods inflation $\pi_t =$ wage inflation, π_t^w, minus TFP growth.

Government Fiscal Policy

- Let be B_t the amount of gov. bonds. The government budget constraint:

$$B_t = (1 + r_{t-1})B_{t-1} + G_t - T_t$$

- Iterating and imposing a no-Ponzi scheme, we get the gov. intertemporal BC:

$$(1 + r_{t-1})B_{t-1} = \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1 + r_s} \right) (T_t - G_t)$$

- Aggregate tax revenue adjusts through τ_t according to:

$$T_t = \int \left[\frac{W_t}{P_t} e_{it} n_{it} - \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it} \right)^{1-\lambda} \right] di$$

Monetary Policy

- Assume no monetary shocks and that monetary policy follows a **real rate rule**.

- Equivalent to Taylor rule with coefficient, $\phi_\pi = 1$, on inflation.

$$r_t = r_{ss} + \varepsilon_t \qquad \Longleftrightarrow \qquad i_t = r_{ss} + \pi_t + \varepsilon_t$$

- Since there are no monetary shocks, $\varepsilon_t = 0$, by the Fisher equation implies a constant interest rate equal to the flexible-price steady-state interest rate r_{ss}.

$$r_t = i_t - \pi_t \qquad \Longrightarrow \qquad r_t = r_{ss} \qquad \text{for all } t = 0, ...\infty$$

- Intuitively, the nominal interest rates rise exactly enough to offset the (expected) inflation.
  - It brings tractability and allows the analysis to focus on forces orthogonal to monetary policy.

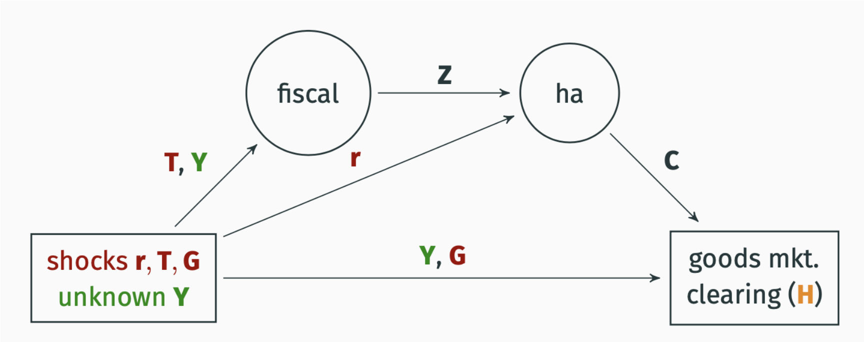
Equilibrium

- Given initial nominal wage W_{-1}, gov. debt B_{-1}, distribution $\Psi_{-1}(\{a^j, e\})$, and exogenous sequences for fiscal policy $\{G_t, T_t\}$, equilibrium is a path for prices, aggregates and individual allocations s.t agents maximize, policies are satisfied and **goods and bond market clear**:

$$G_t + \underbrace{\int c_t(\{a^j\}, e) d\Psi_t}_{C_t} = Y_t$$

$$\sum_j \int a^j d\Psi_t = B_t$$

Equilibrium: DAGs



- Goods mkt. clearing: $H \equiv C + G - Y$

Aggregate Consumption Function

- Let Z_t be the aggregate after-tax income:

$$Z_t \equiv \int z_{it} di = \tau_t N_t^{1-\lambda} \int e_{it}^{1-\lambda} di$$

- Individual after-tax income is a fraction of the aggregate:

$$z_{it} = \frac{e_{it}^{1-\lambda}}{\int e_{st}^{1-\lambda} ds} Z_t$$

- Given that r is constant and z_{it} is proportional to aggregate income Z_t, the individual policy rules $\{c_t, a_t^j\}$ is **entirely determined by the sequence of $\{Z_t\}$**.

Aggregate Consumption Function

- The **aggregate consumption function** is the aggregate of individual policies:

$$\int_i c_{it} di = C_t(\{Z_s\}) = C_t(\{Y_s - T_s\})$$

- Note that C_t depends on the **sequence** of $\{Z_s\}_{s=0}^{\infty} \Rightarrow C_t(Z_0, Z_1, ...)$.

- C_t encapsulates the complex interactions between heterogeneity, macroeconomic aggregates, and wealth distribution.
  - It is **forward-looking** (from the Euler Equation).
  - It also is **backward-looking** (from the distribution and HH budget constraint).

- The consumption function will be different for each model (HA, RA, TA).

The Keynesian Cross

- The consumption function implies a **Keynesian-Cross** type of equation:

$$Y_t = C_t(\{Y_s - T_s\}) + G_t.$$

- Reminds you something? Recall your undergrad macro 1:

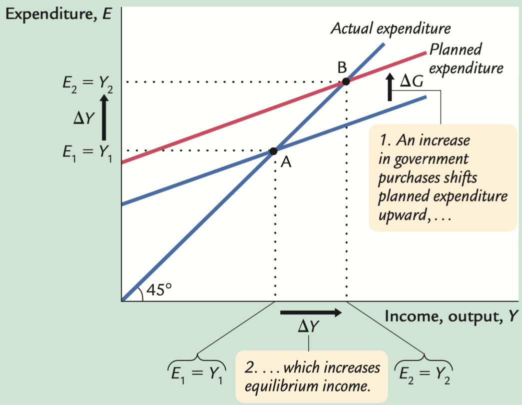
$$Y = C(Y - T) + G \qquad \text{where} \qquad C(Y - T) = c_0 + mpc \times (Y - T).$$

- The difference is that the power of fiscal policy depends not only on the current marginal propensity to consume but on the future and past mpc's as well.

  $\Longrightarrow$ Intertemporal mpc $(iMPC)$!

Undergraduate Keynesian Cross

figure 10-5



An Increase in Government Purchases in the Keynesian Cross An increase in government purchases of ΔG raises planned expenditure by that amount for any given level of income. The equilibrium moves from point A to point B, and income rises from Y_1 to Y_2. Note that the increase in income ΔY exceeds the increase in government purchases ΔG. Thus, fiscal policy has a multiplied effect on income.

- The **intertemporal Keynesian cross** is the same... just in vectors!

Intertemporal MPCs

- What is the effect of fiscal policy (i.e., G_t and T_t) on output? The goods mkt. clearing contains all the complexity of GE.

- Totally differentiating, we get the first-order response of output to changes in fiscal policy:

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t}{\partial Z_s}(dY_s - dT_s)$$

- The **intertemporal MPCs** represent how much consumption at t responds to a change in income at s:

$$M_{t,s} \equiv \frac{\partial C_t}{\partial Z_s}$$

- Since BC holds, all income is eventually spent, which implies: $\sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^{t-s}} = 1$. Proof

The Intertemporal Keynesian Cross

- Collect all the $M_{t,s}$ as the elements of a matrix $\mathbf{M}_{T\times S}$. Let the vectors represent the time sequences: $d\mathbf{Y} \equiv (dY_0, dY_1, ...)'$ (similarly for $d\mathbf{G}$ and $d\mathbf{T}$).

- If the response of output $d\mathbf{Y}$ to a fiscal policy shock $\{d\mathbf{G},\ d\mathbf{T}\}$ exists, it solves the **intertemporal Keynesian cross**:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- Let $\mathcal{M}$ some linear map that ensures $dY_t \to 0$ as $t \to \infty$, the solution is

$$d\mathbf{Y} = \mathcal{M}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$$

  There may be several $\mathcal{M}$ that solve for the linear map (indeterminacy). They restrict attention to $\lim_{\to\infty} dY_t \to 0$.

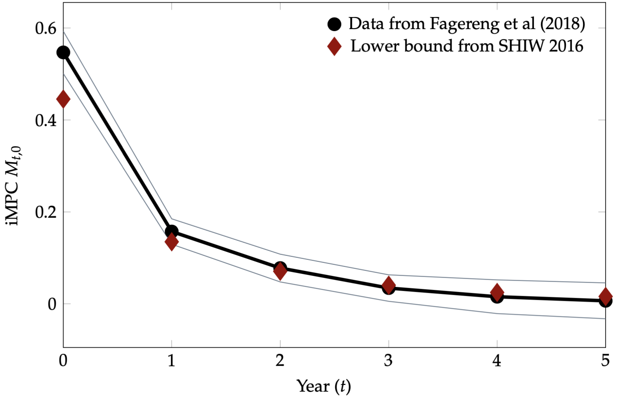
The Intertemporal Keynesian Cross

- The iMPC matrix is a **sufficient statistic**:
  - The entire complexity of the model is in $\mathbf{M}$.
  - The response of Y to fiscal policy shocks is in $\mathbf{M}$.
- There is a "correct" $\mathbf{M}$ out there in the data from the real world (it is just very hard to measure).
- It was possible to derive the "simple" intertemporal Keynesian cross given the many simplified assumptions.
  - Extensions: alternative tax incidence, durable goods, investment.
  - Limitations: passive monetary policy, sticky prices.

Which model matches the iMPC?

- Data on iMPC is hard to get. We usually only observe the first column $M_{t,0}$ for $t = 0, 1 \ldots$.

Figure 1: iMPCs in the Norwegian and Italian data.



The iMPCs of the Representative Agent Model

- Suppose $\beta(1+r)=1$, iterating the budget constraint and using the EE, the consumption function of the RA is:

$$C_t = (1-\beta)\sum_{s=0}^{\infty} \beta^s Z_s + r a_{-1}.$$

Proof

- Since $M_{t,s} = \frac{\partial C_t}{\partial Z_s} = (1-\beta)\beta^s$, the iMPC matrix is:

$$\mathbf{M}^{RA} = \begin{bmatrix} 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \dots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \dots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The iMPCs of the Two Agent Model

- A fraction μ are hand-to-mouth agents (HTM), $1-\mu$ are permanent income agents (PIH).
- Consumption function of each type of agent:

$$c_t^{PIH} = (1-\beta)\sum_{s=0}^{\infty}\beta^s Z_s + ra_{-1}, \qquad \text{and} \qquad c_t^{HTM} = Z_t$$

- Aggregate consumption function: $C_t = (1-\mu)c_t^{PIH} + \mu c_t^{HTM}$.
- The iMPC matrix is just a linear combination of both:

$$\mathbf{M}^{TA} = (1-\mu)\mathbf{M}^{RA} + \mu\mathbf{I}$$

- An useful extension is to introduce bonds/wealth in the utility function to mimic incomplete markets (TABU).

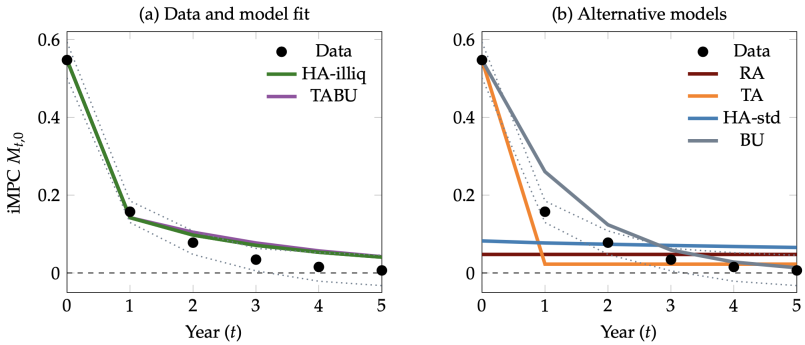
Which model matches the iMPC?

Table 1: Calibrating the benchmark models.

Parameters	Description	Values					
		HA-illiq	RA	TA	HA-std	BU	TABU
ν	Elasticity of intertemporal substitution	0.5	(same across all models)				
ϕ	Frisch elasticity of labor supply	1	(same across all models)				
r	Real interest rate	5%	(same across all models)				
λ	Retention function curvature	0.181	(same across all models)				
G/Y	Government spending to GDP	0.2	(same across all models)				
A/Z	Wealth to after-tax income ratio	8.2	(same across all models)				
β	Discount factor	0.80	0.95	0.95	0.92	0.90	0.90
B/Z	Liquid assets to after-tax income	0.26	8.2	8.2	8.2	8.2	8.2
$\underline{a}$	Borrowing constraint	0			0		
μ	Share of hand-to-mouth households			52%			36%

Which model matches the iMPC?

Figure 2: iMPCs in the Norwegian data and several models.



- HA with low liquidity (tight borrowing constraints or multiple illiquid assets) and TABU fit the data better.

Fiscal Policy

- Focus on two types of multipliers:
  - **Impact Multiplier:** dY_0/dG_0, and **Cumulative Multiplier:** $\frac{\sum_{t=0}^{\infty}(1+r)^{-1}dY_t}{\sum_{t=0}^{\infty}(1+r)^{-1}dG_t}$.
- Benchmark: Balanced budget multiplier $d\mathbf{G} = d\mathbf{T}$.
  - **Fiscal multiplier is always one:** $d\mathbf{Y} = d\mathbf{G}$.
  - Proof is trivial, $d\mathbf{Y} = d\mathbf{G}$ is the only solution of the iKC:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

  - **Intuition**: the increase in pretax income exactly offsets the increase in taxes for every household at every date and state.

Deficit Financed Fiscal Policy

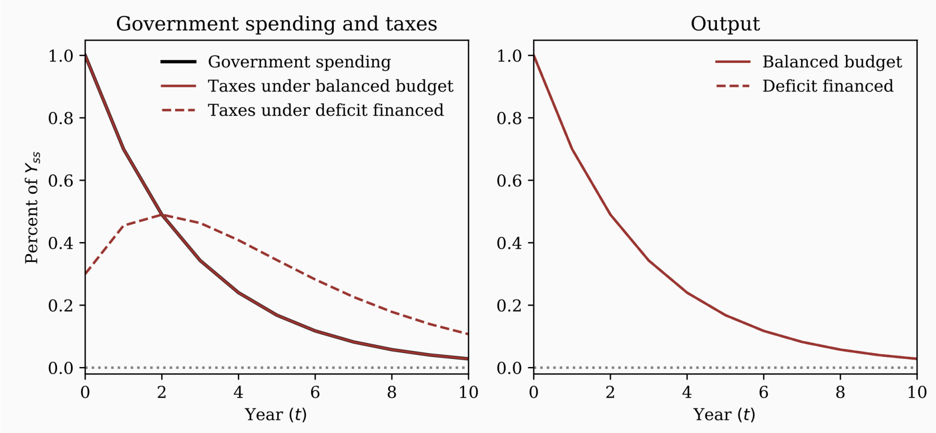
- Suppose a change in fiscal policy is financed with a deficit, i.e $d\mathbf{G} \neq d\mathbf{T}$. Then:

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T})}_{d\mathbf{C}}$$

- The change in consumption $d\mathbf{C}$ depends on the path of primary deficits $(d\mathbf{G} - d\mathbf{T})$.
- Crucial **interaction** between the iMPC matrix $\mathbf{M}$ and the primary deficit.
    - Different models have different $\mathbf{M}$.
    - May be worth running a deficit precisely at the time when iMPC is large.

Fiscal Policy in Representative Agent Model

- In the RA, $d\mathbf{Y} = d\mathbf{G}$ irrespective of $d\mathbf{T}$. Impact and cumulative multipliers are equal to 1.
  - **Intuition:** Since Ricardian Equivalence holds any policy is equivalent to a balanced budget.
  - This result may break with other types of monetary rules, ZLB, etc (Woodford, 2011).



Fiscal Policy in Two Agent Model

- In the TA model, the iKC equation is given by (see paper):

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu}(d\mathbf{G} - d\mathbf{T})$$

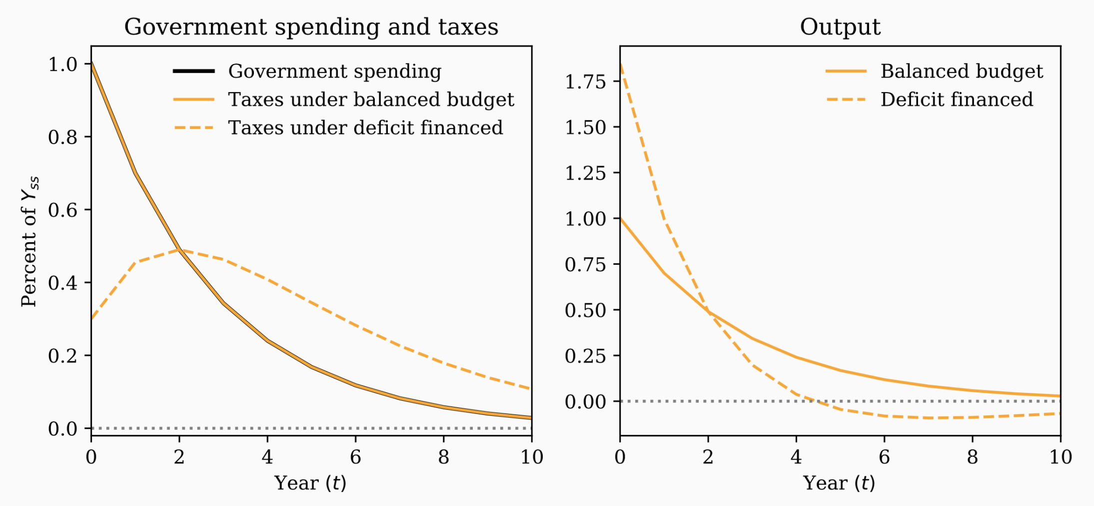
- Only **current deficit** matters.
  - The impact multiplier is a function of the share of HTM agents and the current deficit

$$\frac{1}{1-\mu} - \frac{\mu}{1-\mu}\frac{dT_0}{dG_0}$$

  - Cumulative multiplier is equal to one since consumption declines as soon as deficits are turned into surpluses.

- Model behaves remarkably similarly to static (undergrad) Keynesian cross.

Fiscal Policy in Two Agent Model

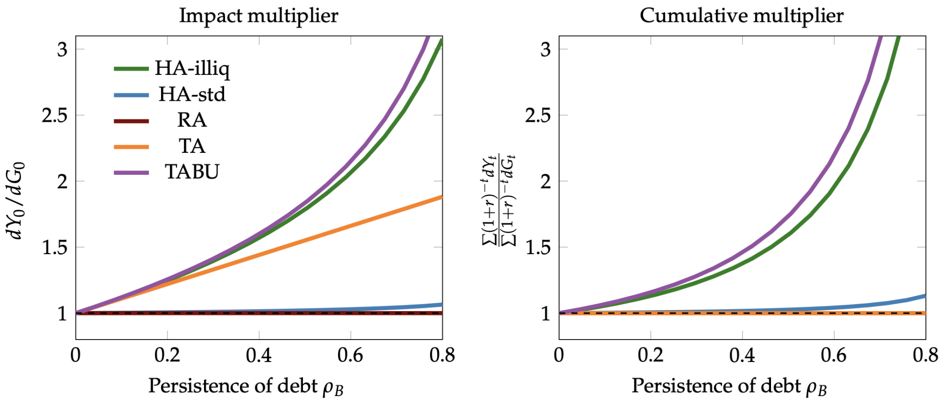


Fiscal Policy in the Benchmark Cases

- Suppose that government spending declines at a rate, $dG_t = \rho_G^t$.
- Taxes are chosen such that the path of public debt is given by: $dB_t = \rho_B(dB_{t-1} - dG_t)$.
  - Greater $\rho_B > 0$ leads to greater deficit.
  - If $\rho_B = 0$ policy keeps a balanced budget.
- Fiscal policy in HA agents can generate (deficit-financed) cumulative multipliers well above 1.
  - Intuition from zero-liquidity HA model (see notes).
  - Multiplier is a combination of the TA model, but with additional anticipatory and backward-looking terms.

Fiscal Policy in the Benchmark Cases

Figure 4: Multipliers across the benchmark models.



- The higher ρ_B, the higher is the multiplier.

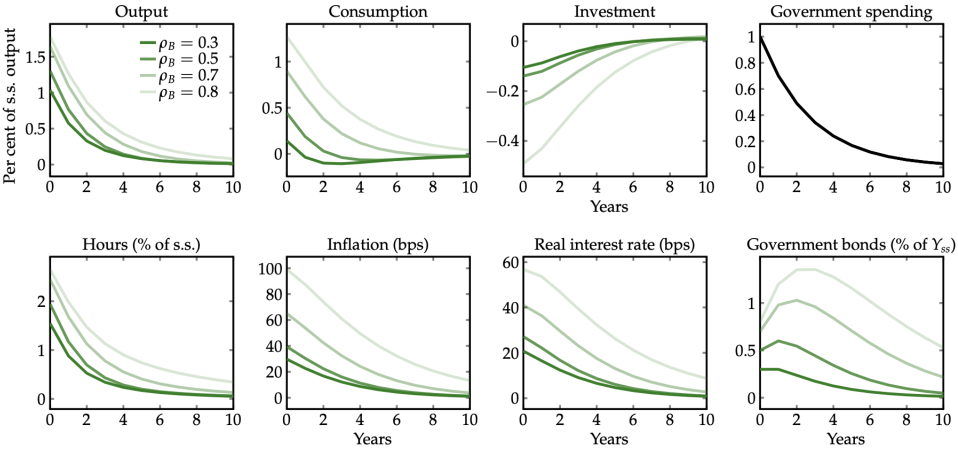
Fiscal Policy in the Quantitative Model

- Benchmark models kept the “supply side” simple to focus on iMPC.

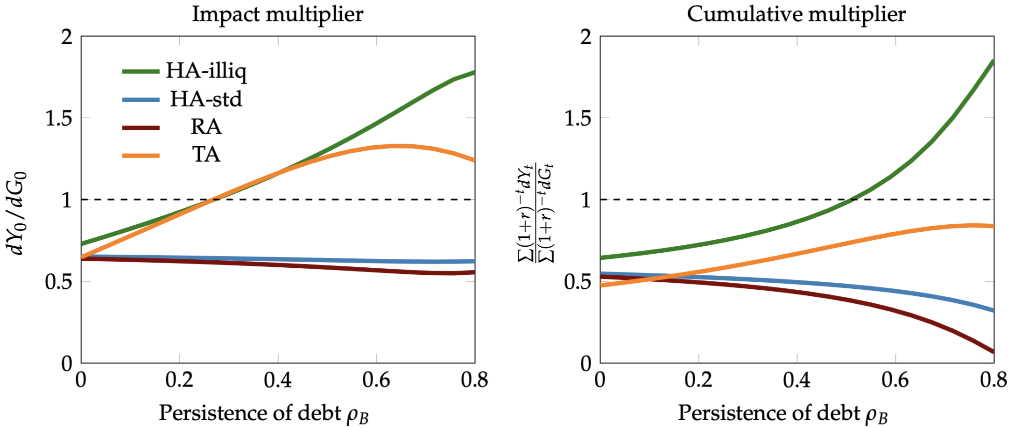
- Compare with the full quantitative model:
  - Capital adjustment shocks;
  - Sticky prices;
  - Portfolio decision;
  - Monetary policy following a Taylor rule.

- The magnitude is smaller, but similar results hold (deficit-financed fiscal policy is stronger).
  - The supply side crowds out part of the effect $\Rightarrow \uparrow r$ and $\downarrow I$.

Fiscal Policy in the Quantitative Model



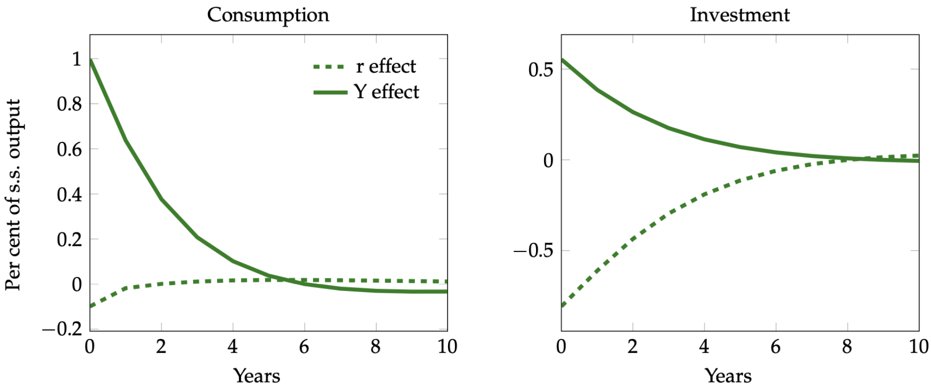
Multiplier in the Quantitative Model



- **Valerie Ramey:** multiplier for temporary deficit-financed spending is "probably between 0.8 and 1.5".

Decomposing the Responses

Figure 6: Decomposing the consumption and investment responses



Extensions and Other Shocks

- Generalization of the iKC allow to separate the effect of **public** and **private** deficit:

$$d\mathbf{Y} = \underbrace{d\mathbf{G} - d\mathbf{T}}_{\text{public deficits}} + \underbrace{(\mathbf{I} - \mathbf{M})d\mathbf{T} + \partial\mathbf{C}}_{\text{PE private deficits}} + \mathbf{M}d\mathbf{Y}$$

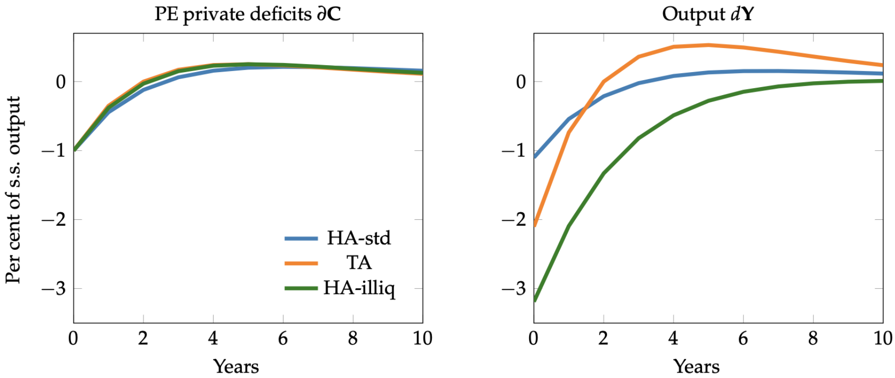
  where $\partial\mathbf{C}$ is the direct consumption effect of a shock to HH, prior to any GE feedback.

- The **PE private deficits** combines:
  - Net HH spending $(\mathbf{I} - \mathbf{M})d\mathbf{T}$ from change in taxes;
  - Direct effect $\partial\mathbf{C}$ of the shock on HH consumption.

- Illustrate with two examples: **deleveraging shock** and **lump-sum financed government spending**.

Deleveraging Shock

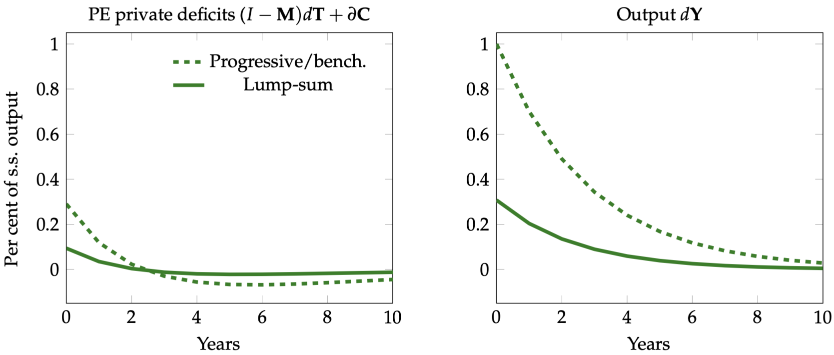
Figure 8: The effects of deleveraging shocks.



- **Deleveraging Shock**: Tightening of borrowing constraint $\underline{a}$.
- The deleveraging shock acts as a reduction of the private deficit and is captured by $\partial \mathbf{C}$.

Fiscal Policy is Less powerful if Financed by Lump-sum Taxes

Figure 9: Comparing two ways to finance government spending: progressive vs. lump-sum taxation.



- Lower PE private deficits on impact under lump-sum ⇒ This taxation targets many constrained households who have little ability to smooth consumption.

Conclusion

- New set of moments captures the GE effects of fiscal policy: **iMPCs**.
- HA with low liquidity matches the iMPCs of the data.
- Balanced-budget fiscal policy is weak even without heterogeneity.
- Deficit-financed fiscal policy is powerful and may have high impact and cumulative multipliers!
- Novel results on distortionary taxation, active monetary policy and others!

Appendix

Sticky Wages: Unions

- Problem of union k:

$$\max_{\{W_{kt+\tau}\}} \sum_{\tau \geq 0} \beta^{t+\tau} \left(\int \{u(c_{it+\tau}) - v(n_{it+\tau})\} d\Psi_{it+\tau} - \frac{\psi}{2} \left(\frac{W_{kt+\tau}}{W_{kt+\tau-1}} - 1 \right)^2 \right)$$

subject to HH budget constraint and $N_{kt} = (W_{kt}/W_t)^{-\epsilon} N_t$ for all t.

- Using the fact that $\partial c_{it}/\partial W_{kt} = \partial z_{it}/\partial W_{kt}$ and $n_{it} \equiv \int_0^1 (W_{kt}/W_t)^{-\epsilon} N_t dk$, F.O.C implies

$$\int \left\{ \frac{\partial z_{it}}{\partial W_{kt}} u'(c_{it}) + \frac{\epsilon}{W_{kt}} \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} N_t v'(n_{it}) \right\} d\Psi_{it} ...$$

$$... - \psi \left(\frac{W_{kt}}{W_{kt-1}} - 1 \right) \frac{1}{W_{kt-1}} + \beta\psi \left(\frac{W_{kt+1}}{W_{kt}} - 1 \right) \frac{W_{kt+1}}{W_{kt}} \frac{1}{W_{kt}} = 0$$

Sticky Wages: Unions

$$\psi\left(\frac{W_{kt}}{W_{kt-1}}-1\right)\frac{W_{kt}}{W_{kt-1}} = W_{kt}\int\left\{\frac{\partial z_{it}}{\partial W_{kt}}u'(c_{it}) + \frac{\epsilon}{W_{kt}}\left(\frac{W_{kt}}{W_t}\right)^{-\epsilon} N_t v'(n_{it})\right\} d\Psi_{it}\ldots$$

$$\ldots + \beta\psi\left(\frac{W_{kt+1}}{W_{kt}}-1\right)\frac{W_{kt+1}}{W_{kt}}$$

- Using $\pi_t^w = W_{kt}/W_{kt-1} - 1$ and $\partial z_{it}/\partial W_{kt} \cdot W_{kt} = \partial z_{it}/\partial n_{it} \cdot (1-\epsilon)N_{kt}$

$$\pi_t^w(1+\pi_t^w) = \frac{1}{\psi}W_{kt}\int\left\{\frac{\partial z_{it}}{\partial W_{kt}}u'(c_{it}) + \frac{\epsilon}{W_{kt}}N_{kt}v'(n_{it})\right\} d\Psi_{it} + \beta\pi_{t+1}^w(1+\pi_{t+1}^w)$$

$$\pi_t^w(1+\pi_t^w) = \frac{\epsilon}{\psi}\int N_{kt}\left\{v'(n_{it}) - \frac{(\epsilon-1)}{\epsilon}\frac{\partial z_{it}}{\partial n_{it}}u'(c_{it})\right\} d\Psi_{it} + \beta\pi_{t+1}^w(1+\pi_{t+1}^w)$$

and by symmetry in eq. $n_{it} = N_{kt} = N_t$ and $W_{kt} = W_t$. Back

All income is eventually spent

- Iterating the BC of an arbitrary agent forward (and imposing a NPG):

$$c_0 + a_0 = (1 + r_{-1})a_{-1} + z_0 \qquad \Rightarrow \qquad \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} c_t = (1 + r_{t-1})a_{-1} + \sum_{s=0}^{\infty} \frac{1}{(1+r)^t} z_t$$

- Aggregating all agents:

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} C_t(\{Z_s\}) = (1 + r_{t-1})a_{-1} + \sum_{s=0}^{\infty} \frac{1}{(1+r)^t} Z_t$$

- Taking the derivatives with respect to Z_s:

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} M_{t,s} = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \qquad \Leftrightarrow \qquad \sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^{t-s}} = 1. \quad \square$$

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Consumption Function of RA Model

- Since $\beta(1+r)=1$, the EE $c_t^{-\sigma}=\beta(1+r)c_{t+1}^{-\sigma} \implies c_t = c_{t+1} = c_{t+s}$ for all $s=0,1\ldots$
- From the budget constraint:

$$c_t + a_t = (1+r_{t-1})a_{t-1} + z_t \qquad \Rightarrow \qquad \beta c_t + \beta a_t = a_{t-1} + \beta z_t$$

- Iterating the BC at $t=0$ forward (and imposing a NPG):

$$c_0 + a_0 = (1+r_{-1})a_{-1} + z_0 \qquad \Rightarrow \qquad \sum_{s=0}^{\infty}\beta^s c_s = (1+r_{t-1})a_{-1} + \sum_{s=0}^{\infty}\beta^s z_s$$

- Since $c_0 = c_s = C_t$, $z_s = Z_s$ and $(1-\beta)(1+r_{-1}) = r_{-1}$:

$$\frac{C_t}{1-\beta} = \sum_{s=0}^{\infty}\beta^s z_s + (1+r_{t-1})a_{-1} \qquad \Rightarrow \qquad C_t = (1-\beta)\sum_{s=0}^{\infty}\beta^s Z_s + ra_{-1}. \quad \square$$

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