Macroeconomics I
Problem Set 7

1. **(Comparative statics in a search and matching model)**. Use the baseline search and matching model (with Cobb-Douglas matching function) seen in class to analyze what happens to wage, tightness, and unemployment, in the steady state when:

   (a) There is an increase in unemployment benefits, b.

   (b) There is an increase in separation rate, σ.

   (c) There is an increase in bargaining power of the workers, γ.

   (d) There is a decrease in matching efficiency, χ.

2. **(Efficiency and the Hosios condition)**. Consider the search and matching model, evaluated in the steady-state, given by:

$$\text{Wage curve:} \quad w = (1-\gamma)b + \gamma z + \gamma\theta\kappa$$

$$\text{Job creation:} \quad \frac{\kappa}{\beta\lambda_f(\theta)} = \frac{z-w}{1-\beta(1-\sigma)}$$

   (a) Find the condition that determines the equilibrium level of tightness in steady-state.

   (b) Consider the social planner's dynamic problem of this economy

$$\max_{\{v_t, u_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (z_t(1-u_t) + bu_t - \kappa v_t)$$
$$s.t. \quad u_{t+1} - u_t = \sigma(1-u_t) - \chi u_t^{\eta} v_t^{1-\eta}$$

   Find the first-order condition and interpret it.

   (c) Evaluate the social planner's first-order condition in steady-state. What condition has to be satisfied for the decentralized equilibrium to be efficient?

3. **(Random search in the public and private sector)**. Consider the following search and matching model. The labor force consists of a continuum of individuals indexed by $j \in [0,1]$. In each period, a proportion u_t of the individuals are unemployed, a proportion l_t^g are working in the public sector and a proportion l_t^p are working in the private sector, such that

$$l_t^g + l_t^p + u_t = 1$$

The public sector wage is w_t^g and the private sector wage is w_t^p. The new matches created in this economy are determined by a Cobb-Douglas matching function:

$$m_t^{all} = \chi(u_t)^\eta (v_t^{all})^{1-\eta},$$

where $v_t^{all} = v_t^g + v_t^p$ is the total number of vacancies in the economy which is the sum of public and private sector vacancies. The parameter η is the matching elasticity with respect to unemployment, and χ is the matching efficiency. We assume that search is random (workers cannot direct their search to any specific sector). This implies that the new matches in each sector are proportional to the sector share of vacancies:

$$m_t^g = \frac{v_t^g}{v_t^{all}} m_t^{all} \qquad \text{and} \qquad m_t^p = \frac{v_t^p}{v_t^{all}} m_t^{all}.$$

The wage in the private sector is determined by Nash Bargaining, while the wage in the public sector is a policy variable chosen by the government. The rest of the model is standard: workers discount their linear utility by $\beta \in (0, 1)$.

(a) Write down the public and private employment law of motion, knowing that jobs are destroyed at constant fractions σ^g and σ^p, potentially different across sectors.

(b) Write down the vacancy filling probabilities in the two sectors q_t^p and q_t^g, and the job-finding rates in the two sectors f_t^p and f_t^g.

(c) Denote the unemployed benefit as b. Write down the Bellman equations for the value of being unemployed, U_t, employed in the private sector, W_t^p, and employed in the public sector, W_t^g. Briefly explain them.

(d) In this model, how does the public sector wage affect the labor market equilibrium? (don't do any derivations, you can show it in a graph).

(e) Can you conjecture the role of public wages to achieve the efficient allocation when the Hosios condition is not satisfied?