# Macroeconomics I
# Problem Set 6

1. **(OLG with Government).** Consider the standard OLG model where the agent works when young and consumes savings when old. The utility of an agent born in $t$ is:

$$\ln(c_t^1) + \beta \ln(c_{t+1}^2).$$

The government taxes the young via lump-sum taxation, $\tau_t$. Suppose $\delta = 0$. The budget constraints of the agent when young and old for all generations $t \geq 1$ are:

$$c_t^1 + s_t \leq w_t - \tau_t$$
$$c_{t+1}^2 \leq (1 + r_{t+1})s_t$$

The government uses taxation to finance its fiscal policy. Suppose the budget is balanced every period, i.e., the government's budget constraint for all $t \geq 1$ is:

$$\tau_t L_t = G_t \equiv g_t L_t,$$

where $G_t > 0$ is the aggregate spending (and is exogenous), and $g_t$ is spending per effective unit of labor. The production function is: $Y_t = K_t^\alpha L_t^{1-\alpha}$, with $\alpha \in (0, 1)$ and the population grows $L_t = (1 + n)^t L_0$ where $L_0 = 1$. The initial old generation has $K_0 > 0$ given.

(a) Solve the households' problem. Find $s_t$ in terms of the parameters, prices, and tax $\tau_t$.

(b) Write down the equilibrium conditions in the goods market and in the asset market.

(c) Suppose the prices are given by $r_t = \alpha k_t^{\alpha-1}$ and $w_t = (1-\alpha)k_t^\alpha$ for all $t$, where $k_t \equiv K_t/L_t$. Find an equation that describes the evolution of the equilibrium capital in the model ($k_{t+1}$ in terms of $k_t$ and parameters). How does an increase in government spending per worker alter the capital accumulation dynamics?

(d) Suppose $n = 0$ (for simplicity). In the discrete-time neoclassical growth model with infinitely-lived dynasties and fiscal policy financed by lump-sum taxation, the Euler equation is given by:

$$c_{t+1} = c_t[\beta(1 + r_{t+1})] \qquad \forall t.$$

Find the $k_t$ in the steady state in the neoclassical growth model. How does the level of government spending per effective unit of labor, $g$, affect capital in the steady state in the neoclassical growth model? Compare with the OLG model.

2. **(Capital Utilization a la Burnside and Eichenbaum (1996)).** Consider the RBC model with variable capital utilization. The utility of the representative family is standard and given by:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \theta \frac{N_t^{1+\phi} - 1}{1+\phi} \right)$$

where $0 < \beta < 1$, $C_t$ is consumption, $N_t$ is the labor time. We will solve for the optimal equilibrium allocations.

The social planner is subject to the economy's resource constraints and respects the capital motion law. He chooses consumption, investment, household labor, and capital utilization, $u_t$, to maximize the utility of the representative family subject in all periods to:

$$\begin{aligned} Y_t &= C_t + I_t \\ Y_t &= Z_t (u_t K_t)^{\alpha} N_t^{1-\alpha} \\ K_{t+1} &= (1 - \delta_f(u_t))K_t + I_t \end{aligned}$$

where $\delta_f(u_t)$ is a function:

$$\delta_f(u_t) = \frac{\delta}{\psi} u_t^{\psi}, \qquad \psi > \alpha.$$

Finally, $Z_t$ follows a stationary AR(1) process.

(a) Solve the central planner's problem. Which conditions are *intratemporal* and which conditions are *intertemporal*?

(b) Use the FOC of $u_t$ and show that the production function can be written as:

$$Y_t = \left(\frac{\alpha}{\delta}\right)^{\frac{\alpha}{\psi-\alpha}} Z_t^{\frac{\psi}{\psi-\alpha}} K_t^{\frac{\alpha(\psi-1)}{\psi-\alpha}} N_t^{\frac{\psi(1-\alpha)}{\psi-\alpha}}.$$

(c) Explain intuitively how the introduction of capital utilization $u_t$ alters the amplification and persistence of the model. In particular, answer how a positive shock in $Z_t$ changes $N_t$ and $Y_t$. How does this depend on the elasticity of capital utilization $\psi$?[1]

3. **(Money in Utility Function).**[2] Consider the following RBC model with money in the utility function. Households value money in their utility (you can interpret this as a need for money for transactions or liquidity preference). The utility is given by:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \theta \frac{N_t^{1+\phi} - 1}{1+\phi} + \frac{(M_t/P_t)^{1-\nu} - 1}{1-\nu} \right)$$

[1] Hint: observe the production function above. Compare the elasticity of production with respect to the quantity of hours worked in the case with variable utilization and without variable utilization. How do wages respond to a shock $Z_t$?

[2] Based on Walsh (2010, chap. 2)

where $0 < \beta < 1$, $C_t$ is consumption, $N_t$ is the labor time, $P_t$ is the price of the final good (in units of money), and $M_t/P_t$ is the quantity of real money holdings of the household (or real balances). Households choose how much to consume, $C_t$, how much to work, $N_t$, how much money to hold, $M_t$, how much capital to invest $K_{t+1}$, and how many bonds to buy $B_{t+1}$:

The household's budget constraint is:

$$P_tC_t + P_t(K_{t+1} - (1-\delta)K_t) + B_{t+1} + M_t = P_tw_tN_t + P_t\hat{r}_tK_t + (1+i_{t-1})B_t + M_{t-1} + P_tT_t$$

Where $i_t$ is the nominal interest rate paid on bonds and $T_t$ is a lump-sum transfer paid by the government. Government transfer is financed via money printing:

$$P_tT_t = M_t - M_{t-1}.$$

Production is standard and follows a Cobb-Douglas function: $Y_t = K_t^{\alpha}N_t^{1-\alpha}$, where $\alpha \in (0,1)$. The household is subject to a no-Ponzi scheme constraint. Define $P_{t+1}/P_t \equiv 1+\pi_t$ where $\pi_t$ is the inflation rate of the economy.

(a) Describe and solve the consumer's problem. Write the demand for real balances, $M_t/P_t$, as a function of $C_t$ and $i_t$. How does it depend on $i_t$ and $C_t$?

(b) Find the Fisher equation $i_t = \mathbb{E}_t r_{t+1} + \pi_{t+1}$, where $r_t \equiv \hat{r}_t - \delta$.

(c) Suppose that in the steady state the money growth rate is $M_{t+1}/M_t = 1+\mu$ and that real balances are constant in the steady state $M_t/P_t = M_{t+1}/P_{t+1} = m$. Find a system of (eight) equations that solves the problem in the steady state for the variables: $(r, w, K, N, C, i, m, T)$.

(d) Suppose an increase in the money growth rate: $\mu$. How does this affect the variables in the steady state? How do changes in $M_t$ affect the price level and other endogenous variables? What would happen if agents expected an increase in the money supply in $t+1$?