PhD in Business Economics - Insper
Macroeconomics 1
Professor: Tomás Martinez
Midterm 2026

Instructions:

- Write your name on the first page of the exam.
- The exam consists of 3 questions totaling 100 points.
- The duration of the exam is 2 hours.
- It is allowed to consult any material. It is not allowed to communicate with other individuals or to use artificial intelligence.
- Keep the exam organized and the pages numbered.
- If something in the question is unclear, write the assumptions you deem necessary to have a well-defined problem and proceed.
- If you get stuck on a specific part of a question, remember that you may consider the result of that part as given and continue to answer the other parts.

Questions

1. **(Solow Model with Intermediate Inputs – 40 points).** Consider the following economy in discrete time. Aggregate output is produced according to

$$Y_t = K_t^{\alpha}(M_t L_t)^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where K_t is aggregate capital, L_t is labor, and M_t is an intermediate input per worker. The intermediate input is produced according to

$$M_t = B_t k_t^{\gamma}, \qquad k_t \equiv \frac{K_t}{L_t}, \qquad \gamma > 0,$$

where B_t is an exogenous productivity term. Population evolves according to $L_{t+1} = (1+n)L_t$, $n > 0$, and capital accumulates according to $K_{t+1} = sY_t + (1-\delta)K_t$, $0 < s < 1$, $0 < \delta < 1$.

(a) (10 points) Derive the aggregate production function in terms of K_t, L_t, and B_t. Then derive the production function in per-worker terms, i.e. express output per worker $y_t \equiv Y_t/L_t$ as a function of capital per worker $k_t \equiv K_t/L_t$ and B_t.

Solution: Since $M_t = B_t k_t^{\gamma} = B_t(K_t/L_t)^{\gamma}$, we have $M_t L_t = B_t K_t^{\gamma} L_t^{1-\gamma}$. Substituting into the production function:

$$Y_t = K_t^{\alpha}\left(B_t K_t^{\gamma} L_t^{1-\gamma}\right)^{1-\alpha} = B_t^{1-\alpha} K_t^{\alpha+\gamma(1-\alpha)} L_t^{(1-\gamma)(1-\alpha)}.$$

Define $\phi \equiv \alpha + \gamma(1-\alpha)$. Dividing by L_t gives the per-worker production function:

$$y_t = B_t^{1-\alpha} k_t^{\phi}.$$

(b) (15 points) Suppose $B_t = B$ is constant over time. Derive the steady-state level k^*. Under what condition does a unique positive steady state exist? Provide intuition for your answer.

Solution: Dividing $K_{t+1} = sY_t + (1-\delta)K_t$ by $L_{t+1} = (1+n)L_t$ and using $y_t = B^{1-\alpha}k_t^{\phi}$:

$$k_{t+1} = \frac{sB^{1-\alpha}k_t^{\phi} + (1-\delta)k_t}{1+n}.$$

Setting $k_{t+1} = k_t = k^*$, the condition $(n+\delta)k^* = sB^{1-\alpha}(k^*)^{\phi}$ gives

$$k^* = \left(\frac{sB^{1-\alpha}}{n+\delta}\right)^{\frac{1}{1-\phi}}.$$

A unique positive steady state exists if and only if $\phi < 1$, i.e. $\gamma < 1$. If $\phi < 1$, capital accumulation faces diminishing returns and the economy converges to a unique interior steady state. If $\phi = 1$ the model becomes AK-type; if $\phi > 1$ there are increasing returns — in either case no finite stable steady state exists.

(c) (15 points) Now suppose productivity evolves according to $B_{t+1} = (1+g)B_t$, $g > 0$. Show that the economy admits a balanced growth path. Derive the growth rate of capital per worker and output per worker along the BGP. At what rate does aggregate output Y_t grow along the balanced growth path? **Hint**: To show the economy admits a BGP, find the steady state of capital in (rescaled) efficiency units: $\tilde{k}_t \equiv k_t/B_t^\eta$, where η is a function of the parameters of the model. Use the results of the previous item to help you.

Solution: Since B_t is growing, the law of motion for k_t is time-varying and admits no stationary steady state in k_t. The idea is to show that the rescaled variable $\tilde{k}_t \equiv k_t/B_t^\eta$ admits a steady state for an appropriate choice of η. To find η, note that the steady state from part (b) can be rewritten as:

$$k^* = \left(\frac{sB^{1-\alpha}}{n+\delta}\right)^{\frac{1}{1-\phi}} \qquad \Longleftrightarrow \qquad \frac{k^*}{B^{\frac{1-\alpha}{1-\phi}}} = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\phi}},$$

which suggests the guess $\eta \equiv (1-\alpha)/(1-\phi)$. To verify, substitute $k_t = \tilde{k}_t B_t^\eta$ and $k_{t+1} = \tilde{k}_{t+1} B_{t+1}^\eta$ into the law of motion from part (b) and use $B_{t+1}^\eta = B_t^\eta (1+g)^\eta$:

$$\tilde{k}_{t+1}\, B_t^\eta (1+g)^\eta = \frac{sB_t^{1-\alpha}\left(\tilde{k}_t B_t^\eta\right)^\phi + (1-\delta)\tilde{k}_t B_t^\eta}{1+n} = \frac{B_t^{1-\alpha+\eta\phi}\, s\tilde{k}_t^\phi + (1-\delta)\tilde{k}_t B_t^\eta}{1+n}.$$

Dividing both sides by $B_t^\eta (1+g)^\eta$ and using $1-\alpha+\eta\phi = \eta$ (so $B_t^{1-\alpha+\eta\phi} = B_t^\eta$):

$$\tilde{k}_{t+1} = \frac{s\tilde{k}_t^\phi + (1-\delta)\tilde{k}_t}{(1+n)(1+g)^\eta}.$$

This law of motion is stationary in $\tilde{k}_t$ and has the same form as in part (b), so $\tilde{k}_t$ converges to a well-defined steady state $\tilde{k}^*$ under the same condition $\phi < 1$. Along the BGP, $\tilde{k}_t = \tilde{k}^*$ is constant, so $k_t = \tilde{k}^* B_t^\eta$ grows at gross rate $(1+g)^\eta$. For output per worker, using $1-\alpha+\eta\phi = \eta$:

$$y_t = B_t^{1-\alpha} k_t^\phi = (\tilde{k}^*)^\phi\, B_t^{1-\alpha+\eta\phi} = (\tilde{k}^*)^\phi\, B_t^\eta,$$

so y_t also grows at gross rate $(1+g)^\eta = (1+g)^{\frac{1-\alpha}{1-\phi}}$.

Since $Y_t = y_t L_t$ and L_t grows at gross rate $1+n$, aggregate output grows at gross rate

$$(1+n)(1+g)^{\frac{1-\alpha}{1-\phi}}.$$

2. **(Earthquake in the Neoclassical growth model – 20 points).** Consider the standard Neoclassical growth model in continuous time without technological progress. The solution is characterized by the following system of differential equations:

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - \delta - \rho}{\sigma}$$
$$\dot{k}_t = f(k_t) - (\delta + n)k_t - c_t,$$

together with an initial condition $k_0 > 0$ and the transversality condition. Suppose production follows a Cobb-Douglas function, $f(k_t) = k_t^\alpha$, and the economy is initially in the steady state (k_{ss}, c_{ss}).

(a) (15 points) Suppose that at time t_0, an earthquake unexpectedly destroys a fraction $\Delta_0 \in (0,1)$ of the capital stock, so that $k_{t_0} = (1-\Delta_0)k_{ss}$. No parameter of the model changes. Describe the transition dynamics until the economy returns to the steady state. Use a phase diagram and other graphs if necessary.

Solution: Since no parameter changes, the $\dot{c} = 0$ and $\dot{k} = 0$ loci are unchanged. The steady state (k_{ss}, c_{ss}) remains the same. The earthquake is a one-time shock that shifts the *initial condition* of capital, not the model's structure.

At t_0, capital drops discontinuously to $k_{t_0} = (1-\Delta_0)k_{ss} < k_{ss}$. Consumption can jump, and it must do so to place the economy on the (unchanged) saddle path.

At $k = (1-\Delta_0)k_{ss} < k_{ss}$, the saddle path lies *below* c_{ss}. Therefore, consumption **jumps down** at t_0 to the saddle-path value $c_0 < c_{ss}$. The intuition is the standard wealth effect: the destruction of capital reduces lifetime wealth, so agents immediately cut consumption.

- At t_0, both k and c are below their steady-state values: $k_{t_0} = (1-\Delta_0)k_{ss}$ and $c_0 < c_{ss}$.
- Since $f'((1-\Delta_0)k_{ss}) > f'(k_{ss}) = \rho + \delta$, the Euler equation gives $\dot{c}_t/c_t > 0$: consumption is increasing.
- Since (k_{t_0}, c_0) lies below the $\dot{k} = 0$ locus, we have $\dot{k}_t > 0$: capital is accumulating.
- Both k_t and c_t increase monotonically along the saddle path, converging back to (k_{ss}, c_{ss}).

(b) (5 points) Suppose that after the economy has returned to the steady state, the country is hit by another earthquake, this time a smaller one: a fraction $\Delta_1 < \Delta_0$ of the capital stock is destroyed, so that $k_{t_1} = (1-\Delta_1)k_{ss}$. Is the drop in consumption on impact larger or smaller than after the first earthquake? Explain briefly.

Solution: The drop in consumption is **smaller**. Since no parameter changes in either episode, the steady state and the saddle path are identical in both cases. The only difference is the size of the capital destruction: the smaller earthquake places the economy at $k_{t_1} = (1-\Delta_1)k_{ss}$, which is closer to k_{ss} along the saddle path than $k_{t_0} = (1-\Delta_0)k_{ss}$. Since consumption must jump to the saddle path on impact, and the saddle path is continuous and increasing in k, a smaller reduction in capital corresponds to a smaller reduction in consumption. Intuitively, the smaller earthquake destroys less wealth, so the negative wealth effect on consumption is weaker.

3. **(RBC with Investment Adjustment Costs – 40 points).** Consider the following discrete-time RBC economy. The social planner maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\nu}}{1+\nu} \right],$$

subject to the resource constraint

$$c_t + i_t + \frac{\phi}{2} \frac{i_t^2}{k_t} = z_t k_t^{\alpha} n_t^{1-\alpha},$$

the capital accumulation equation

$$k_{t+1} = (1-\delta)k_t + i_t,$$

and the TFP process

$$\ln z_{t+1} = \rho \ln z_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \overset{i.i.d.}{\sim} (0, \sigma_\varepsilon^2),$$

with $k_0 > 0$ and $z_0 = 1$ given. The parameter $\phi \geq 0$ governs the magnitude of investment adjustment costs.

(a) (15 points) Set up the Lagrangian for the planner's problem. Derive all optimality conditions and write the full system of equations characterizing the allocations. Define μ_t as the current-value multiplier of the law of motion of capital.[1]

Solution: Let λ_t and μ_t be the current-value multipliers on the resource constraint and the capital accumulation equation, respectively. The Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\nu}}{1+\nu} + \lambda_t \left(z_t k_t^{\alpha} n_t^{1-\alpha} - c_t - i_t - \frac{\phi}{2} \frac{i_t^2}{k_t} \right) + \mu_t((1-\delta)k_t + i_t - k_{t+1}) \Bigg].$$

FOC for c_t: $\lambda_t = c_t^{-\sigma}$.
FOC for n_t: $\chi n_t^{\nu} = \lambda_t (1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha}$, which using $\lambda_t = c_t^{-\sigma}$ gives

$$\chi n_t^{\nu+\alpha} = c_t^{-\sigma}(1-\alpha) z_t k_t^{\alpha}.$$

FOC for i_t:

$$\mu_t = \lambda_t \left(1 + \phi \frac{i_t}{k_t} \right) = c_t^{-\sigma} \left(1 + \phi \frac{i_t}{k_t} \right).$$

FOC for k_{t+1}:

$$\mu_t = \beta \left[c_{t+1}^{-\sigma} \left(\alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + \frac{\phi}{2} \left(\frac{i_{t+1}}{k_{t+1}} \right)^2 \right) + \mu_{t+1}(1-\delta) \right].$$

The full system consists of equations (1)–(6):

[1] **Hint:** Do NOT substitute i_t in the resource constraint. Write both constraints explicitly in the Lagrangian and take the f.o.c. for both i_t and k_{t+1}.

(1) $c_t + i_t + \dfrac{\phi}{2}\dfrac{i_t^2}{k_t} = z_t k_t^{\alpha} n_t^{1-\alpha}$

(2) $k_{t+1} = (1-\delta)k_t + i_t$

(3) $\chi n_t^{\nu+\alpha} = c_t^{-\sigma}(1-\alpha) z_t k_t^{\alpha}$

(4) $\mu_t = c_t^{-\sigma}(1 + \phi\, i_t/k_t)$

(5) $\mu_t = \beta\left[c_{t+1}^{-\sigma}\left(\alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + \dfrac{\phi}{2}\left(\dfrac{i_{t+1}}{k_{t+1}}\right)^2\right) + \mu_{t+1}(1-\delta)\right]$

(6) $\ln z_{t+1} = \rho \ln z_t + \varepsilon_{t+1}$

In addition to initial $k_0 > 0$ and a transversality condition.

(b) (15 points) Define Tobin's marginal $Q_t \equiv \mu_t / c_t^{-\sigma}$ as the shadow value of installed capital in consumption units (i.e., how much consumption you would give up for one extra unit of installed capital). Show that the optimality condition for i_t implies

$$\frac{i_t}{k_t} = \frac{Q_t - 1}{\phi}.$$

What does $Q_t > 1$ imply for investment? What happens as $\phi \to 0$?

Solution: From the FOC for i_t, $\mu_t = c_t^{-\sigma}(1 + \phi\, i_t/k_t)$. Dividing both sides by $c_t^{-\sigma}$ and using $Q_t \equiv \mu_t / c_t^{-\sigma}$:

$$Q_t = 1 + \phi \frac{i_t}{k_t} \qquad \Longrightarrow \qquad \frac{i_t}{k_t} = \frac{Q_t - 1}{\phi}.$$

Interpretation: The marginal cost of investing one unit is $1 + \phi\, i_t/k_t$ — the unit price of investment goods (1) plus the marginal adjustment cost. Optimality equates this cost to the shadow value of installed capital Q_t. If $Q_t > 1$, installed capital is worth more than its replacement cost, so the planner invests above zero; if $Q_t < 1$, the planner disinvests. For a given Q_t, higher ϕ implies lower investment — adjustment costs act as a tax on investment.

As $\phi \to 0$, the condition collapses to $Q_t = 1$ at all times: with no adjustment costs the shadow price of capital always equals its replacement cost and investment adjusts freely to equate them.

(c) (10 points) Suppose the economy is at the deterministic steady state and at $t = 1$ receives an unexpected positive TFP shock ($\varepsilon_1 > 0$, with $z_t \to 1$ thereafter). Describe briefly the impulse response of investment i_t and c_t. Does investment respond more or less than in the case where $\phi = 0$? What is the implication for the response of c_t?

Solution: A positive TFP shock raises the marginal product of capital, increasing the value of having more capital installed. Since Q_t is a forward-looking (jump) variable, it immediately incorporates the higher future returns: Q_1 jumps up on impact. Via $i_t/k_t = (Q_t - 1)/\phi$, this translates into an immediate increase in investment.

However, investment does not jump to the new steady-state level for two reasons:

- *Adjustment costs:* The marginal cost of investment is increasing in i_t/k_t. It is optimal to spread the adjustment over multiple periods rather than incur large convex costs immediately.
- *Capital dynamics:* As capital gradually accumulates, the marginal product of capital declines back toward its steady-state value, pulling Q_t — and therefore investment — gradually back down.

Compared to $\phi = 0$, investment responds **less** on impact: for the same jump in Q, a higher ϕ implies a smaller $i_t/k_t = (Q_t - 1)/\phi$. The response is also more persistent, as capital accumulates more slowly.

Implication for c_t: Since resources not spent on investment are available for consumption, a smaller investment response on impact leaves more output for consumption. Therefore, c_t rises **more** on impact when $\phi > 0$ than when $\phi = 0$. Adjustment costs effectively smooth investment and shift resources toward consumption in the short run. Over time, as capital builds up more slowly, consumption is slightly lower than it would have been without adjustment costs.