PhD in Business Economics - Insper
Macroeconomics 1
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Final 2026

Instructions:

- Write your name on the first page of the exam.
- The exam consists of 4 questions totaling 105 points (5 extra points).
- The duration of the exam is 3 hours.
- It is allowed to consult any material. It is not allowed to communicate with other individuals or to use artificial intelligence.
- Keep the exam organized and the pages numbered.
- If something in the question is unclear, write the assumptions you deem necessary to have a well-defined problem and proceed.
- If you get stuck on a specific part of a question, remember that you may consider the result of that part as given and continue to answer the other parts.

Questions

1. **(Firm Dynamics with Innovation – 20 points).** Consider a competitive firm that produces output using labor only:

$$y_t = z_t^{1-\alpha} n_t^{\alpha}, \qquad \alpha \in (0,1),$$

where $z_t > 0$ is firm productivity and n_t is labor demand. The wage $w > 0$ is constant and taken as given.

After production takes place, the firm chooses an innovation intensity $p_t \in [0,1]$. Innovation costs $\frac{\kappa}{2} z_t p_t^2$, $\kappa > 0$. Productivity evolves according to

$$z_{t+1} = \begin{cases} \lambda z_t & \text{with probability } p_t, \\ z_t/\lambda & \text{with probability } 1 - p_t, \end{cases} \qquad \lambda > 1.$$

The firm discounts future profits using discount factor $\beta \in (0,1)$.

(a) (5 points) Solve the profit maximization problem and show that profits are proportional to z_t, that is, $\pi(z) = \chi z$, where χ is a positive constant that depends on w and the parameters.

Solution: For a given z, the firm chooses n to maximize $z^{1-\alpha} n^{\alpha} - wn$. The first-order condition is $\alpha z^{1-\alpha} n^{\alpha-1} = w$, which gives

$$n^*(z) = \left(\frac{\alpha z^{1-\alpha}}{w}\right)^{\frac{1}{1-\alpha}} = z\left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}}.$$

Substituting back, static profits are $\pi(z) = \chi z$, where

$$\chi = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} w^{-\frac{\alpha}{1-\alpha}}.$$

(b) (9 points) Write the Bellman equation. Clearly identify the state variable, control variable, feasible set, and return function of the problem.

Solution: The state variable is z. The control variable is $p \in [0,1]$, so the feasible correspondence is $\Gamma = [0,1]$. Using $\pi(z) = \chi z$ from part (a), the return function is

$$r(z,p) = \chi z - \frac{\kappa}{2} z p^2.$$

The Bellman equation is

$$V(z) = \max_{p \in [0,1]} \left\{ \chi z - \frac{\kappa}{2} z p^2 + \beta \Big[p V(\lambda z) + (1-p) V(z/\lambda) \Big] \right\}.$$

(c) (6 points) It can be shown that the value function takes the form $V(z) = Az$ for some constant $A > 0$. Using this result, derive the optimal innovation intensity p^* from the first-order condition of the Bellman equation. Show that it does not depend on z. Assume an interior solution.

Solution: Substituting $V(z) = Az$ into the Bellman equation and dividing both sides by $z > 0$:

$$A = \max_{p \in [0,1]} \left\{ \chi - \frac{\kappa}{2} p^2 + \beta A \big[p\lambda + (1-p)/\lambda \big] \right\}.$$

The first-order condition for an interior optimum is

$$-\kappa p + \beta A\left(\lambda - \frac{1}{\lambda}\right) = 0,$$

so the optimal innovation intensity is

$$p^* = \frac{\beta A}{\kappa}\left(\lambda - \frac{1}{\lambda}\right).$$

Higher λ raises the value of success relative to failure, increasing p^*. Higher κ raises the marginal cost of innovation, reducing p^*.

2. **(OLG with Government Debt – 28 points).** Consider the discrete-time overlapping-generations model with production studied in class. Each generation lives for two periods. When young, a household supplies one unit of labor inelastically, earns wage w_t, pays a proportional labor income tax $\tau_t w_t$, consumes c_t^1, and saves s_t. When old, it consumes $c_{t+1}^2 = (1+r_{t+1})s_t$. Preferences are $u(c_t^1) + \beta u(c_{t+1}^2)$, $\beta \in (0,1)$.

The representative firm has a neoclassical, constant-returns-to-scale technology $Y_t = F(K_t, L_t)$. Population grows at rate n, so $L_{t+1} = (1+n)L_t$. Factor markets are competitive and there is no depreciation.

The government issues one-period debt B_t and finances interest payments entirely with the labor income tax. There is no government spending. Let $b_t \equiv B_t/L_t$ denote debt per worker.

(a) (6 points) Suppose there is a unique steady state with positive $k^* > 0$. Write the household budget constraints, the firm's optimality conditions (in per-worker terms), and the capital market clearing condition in the steady state (you do not need to solve for k^*).

Solution: Writing $k \equiv K/L$, $f(k) \equiv F(k,1)$. The household budget constraints are

$$c^1 + s = (1-\tau)w, \qquad c^2 = (1+r)s.$$

The firm's optimality conditions are

$$r = f'(k), \qquad w = f(k) - kf'(k).$$

Private saving finances both capital and government debt. The capital market clears when

$$s = (1+n)(k+b).$$

(b) (8 points) Derive the government budget constraint in per-worker terms. If debt per worker is constant at $b_t = b$, what labor tax rate τ is required in steady state? Find τ in terms of functional forms, parameters and k^*.

Solution: The government budget constraint is $B_{t+1} = (1+r_t)B_t - \tau_t w_t L_t$. Dividing by L_t and using $L_{t+1} = (1+n)L_t$:

$$(1+n)b_{t+1} = (1+r_t)b_t - \tau_t w_t.$$

Setting $b_{t+1} = b_t = b$ in steady state:

$$(1+n)b = (1+r)b - \tau w \qquad \Rightarrow \qquad \tau = \frac{(r-n)\,b}{w} = \frac{(f'(k^*)-n)\,b}{f(k^*) - k^* f'(k^*)}.$$

The government must raise taxes equal to the excess of interest payments over the natural growth-financed rollover of debt.

(c) (7 points) Explain how a permanent increase in debt b affects the steady-state capital stock k and the steady-state interest rate r.

Solution: From the capital market clearing condition, $s = (1+n)(k+b)$, a higher b crowds out capital for a given level of saving. Moreover, from the government budget constraint, higher b requires a higher tax τ, which reduces disposable income $(1-\tau)w$ and hence saving. Both channels reduce k in steady state:

$$b \uparrow \Rightarrow \; k \downarrow.$$

Since $f''(k) < 0$, a lower capital stock raises the marginal product of capital:

$$k \downarrow \Rightarrow \; r = f'(k) \uparrow.$$

(d) (7 points) Suppose the economy is dynamically inefficient. Explain why public debt may be welfare improving. What is the intuition?

Solution: Dynamic inefficiency occurs when $r < n$: the economy overaccumulates capital relative to the golden rule. In this case, reducing the capital stock raises steady-state consumption per worker. Since public debt crowds out capital (part (c)), it moves the economy toward the golden rule, improving the consumption allocation for all future generations.

The intuition is that when $r < n$, productive capital is so abundant that its social return is lower than the economy's growth rate. Households save "too much" because they cannot commit to transfers to their future selves in old age. Government debt provides an alternative store of value that absorbs excess saving, reduces overaccumulation, and transfers resources from the young (who save) to the old (who hold the debt) — a Pareto improvement across generations.

3. **(Mortensen–Pissarides with Vacancy Tax – 30 points).** Consider the discrete-time Mortensen–Pissarides model with matching function

$$M(u, v) = Au^{\alpha}v^{1-\alpha},$$

where $A > 0$ and $\alpha \in (0, 1)$. Let labor market tightness be $\theta_t \equiv v_t/u_t$, so that the worker finding rate and vacancy filling rate are

$$\lambda_w(\theta_t) = M(1, \theta_t) = A\theta_t^{1-\alpha}, \qquad \lambda_f(\theta_t) = M(\theta_t^{-1}, 1) = A\theta_t^{-\alpha}.$$

Matches separate exogenously with probability $\sigma \in (0, 1)$. A filled job produces output z, an unemployed worker receives flow payoff b, and posting a vacancy costs $(1 + \tau)\kappa$ per period, where $\tau \geq 0$ is a vacancy tax. Assume a stationary equilibrium throughout.

(a) (8 points) Derive the Bellman equations for a filled job, and a vacancy. State the free-entry condition and use it to derive the job creation condition as a function of the output z, the wage w and tightness θ.

Solution: Let J and V denote the value of a filled job and a vacancy, respectively. The Bellman equations are

$$J = z - w + \beta(1 - \sigma)J,$$
$$V = -(1 + \tau)\kappa + \beta\Big[\lambda_f(\theta)J + \big(1 - \lambda_f(\theta)\big)V\Big].$$

In a stationary equilibrium the job value simplifies to

$$J = \frac{z - w}{1 - \beta(1 - \sigma)}.$$

Free entry drives vacancy value to zero, $V = 0$, so the vacancy condition becomes

$$(1 + \tau)\kappa = \beta\lambda_f(\theta)J.$$

Hence, the job creation condition is:

$$\frac{z - w}{1 - \beta(1 - \sigma)} = \frac{(1 + \tau)\kappa}{\beta\lambda_f(\theta)} \qquad \text{(job creation condition)},$$

which pins down θ for a given z and w.

(b) (8 points) Analyze the effect of an increase in τ on labor market tightness θ, the vacancy-filling rate $\lambda_f(\theta)$, the job-finding rate $\lambda_w(\theta)$, and the steady-state unemployment rate u.

Solution: An increase in τ raises the effective vacancy cost $(1 + \tau)\kappa$, making vacancy posting less profitable. For a given J, the job creation condition $\beta\lambda_f(\theta)J = (1 + \tau)\kappa$ can only hold if $\lambda_f(\theta)$ rises. Since $\lambda_f(\theta) = A\theta^{-\alpha}$ is decreasing in θ, this requires θ to fall:

$$\tau \uparrow \Rightarrow \theta \downarrow.$$

The filling rate and finding rate then respond as follows:

$$\lambda_f(\theta) = A\theta^{-\alpha} \uparrow, \qquad \lambda_w(\theta) = A\theta^{1-\alpha} \downarrow.$$

The steady-state unemployment rate satisfies inflows = outflows:

$$\sigma(1-u) = \lambda_w(\theta)\, u \;\Rightarrow\; u = \frac{\sigma}{\sigma + \lambda_w(\theta)}.$$

Since $\lambda_w(\theta)$ falls, steady-state unemployment rises:

$$\tau \uparrow \Rightarrow\; u \uparrow.$$

(c) (8 points) Suppose wages are determined by Nash bargaining with worker bargaining power $\gamma \in (0,1)$, where $\gamma > \alpha$. Is equilibrium vacancy creation too high or too low relative to the planner's allocation? Explain.

Solution: Efficiency in the Mortensen–Pissarides model requires the *Hosios condition*: $\gamma = \alpha$. With Cobb–Douglas matching, α is the elasticity of the matching function with respect to unemployment, which equals the workers' share of match surplus in the planner's solution.

If $\gamma > \alpha$, workers capture a larger share of match surplus than the planner would allocate them. Firms therefore retain too little surplus from a filled job (J is too low), which makes vacancy posting less attractive. As a result, equilibrium vacancy creation is *too low* relative to the social optimum: $\theta < \theta^*$, and unemployment is too high.

(d) (6 points) Can a vacancy tax restore efficiency in this case? If not, what policy would?

Solution: No. When $\gamma > \alpha$, the economy already suffers from *too little* vacancy creation. A vacancy tax raises posting costs further and lowers θ even more, moving the economy farther from the efficient allocation.

To restore efficiency the government must *subsidize* vacancy posting (i.e., set $\tau < 0$), which lowers the effective cost $(1+\tau)\kappa$, raises the return to posting vacancies, and increases θ toward θ^*. More generally:

$$\gamma > \alpha \;\Rightarrow\; \text{vacancy subsidy}, \qquad \gamma < \alpha \;\Rightarrow\; \text{vacancy tax}.$$

The appropriate subsidy/tax rate is the one that equates the decentralized θ to the Hosios-efficient θ^*.

4. **(Sovereign Default – 27 points).** Consider a country with exogenous stochastic endowment Y_t, following a first-order Markov process. The country can borrow or save on international bond markets by choosing next-period debt B_{t+1}. The country faces a pricing schedule reflecting default risk: $q(Y_t, B_{t+1})$ is a continuous, differentiable function, increasing in Y_t and decreasing in B_{t+1}.[1] There is a debt limit $B_t \leq \bar{B}$ for all t.

   Each period, the country first decides whether to default on its outstanding debt B_t. If it defaults, it is excluded from international credit markets forever and consumes its endowment from then on. If it does not default, it repays B_t, chooses B_{t+1}, and consumes

$$C_t = Y_t - B_t + q(Y_t, B_{t+1})B_{t+1}.$$

   Preferences are $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$, $\beta \in (0,1)$.

   (a) (8 points) What is the value for the country once it has defaulted? Write down a value function equation that pins down this value.

   Solution: If the country defaults, it consumes its endowment forever and is excluded from credit markets. Therefore,

$$V_d(Y) = u(Y) + \beta\,\mathbb{E}\big[V_d(Y') \mid Y\big].$$

   Equivalently,

$$V_d(Y) = \mathbb{E}\left[\sum_{s=t}^{\infty} \beta^{s-t} u(Y_s) \,\middle|\, Y_t = Y\right].$$

   (b) (10 points) Bring the problem (before the default decision) into dynamic-programming form. Identify the state variables and controls, the feasible-set correspondence, the return function, and the law of motion for the state. Then write the Bellman equation.

   Solution:

   State variables and controls. The state is (Y, B). The controls are: (i) the default decision $d \in \{0,1\}$; and (ii) next-period debt B', chosen only if the country repays ($d = 0$).

   Feasible-set correspondence.

$$d \in \Gamma_d(Y, B) = \{0, 1\},$$

   and

$$B' \in \begin{cases} \{0\} & \text{if } d = 1, \\ \Gamma_{B'}(Y,B) = \big[B'_{\min}(Y,B),\ \bar{B}\big] & \text{if } d = 0, \end{cases}$$

   where $B'_{\min}(Y,B)$ is the minimum debt consistent with $C \geq 0$, i.e. the unique B' solving $Y - B + q(Y,B')B' = 0$.

   Return function.

$$F(Y,B;d,B') = \begin{cases} u(Y) & \text{if } d = 1, \\ u\big(Y - B + q(Y,B')B'\big) & \text{if } d = 0. \end{cases}$$

   Law of motion. (Y', B'), where Y' follows the Markov transition and B' is the control.

[1] The country receives $q(Y_t, B_{t+1})B_{t+1}$ units of the consumption good at t and promises to repay B_{t+1} units at $t+1$, so the gross interest rate on the bond is $1/q(Y_t, B_{t+1})$.

Bellman equation. Let $V_{nd}(Y, B)$ denote the value of repaying. Then

$$V(Y, B) = \max\big\{V_d(Y),\ V_{nd}(Y, B)\big\},$$

where

$$V_{nd}(Y, B) = \max_{B' \in \Gamma_{B'}(Y,B)} \big\{u\big(Y - B + q(Y, B')B'\big) + \beta\, \mathbb{E}\big[V(Y', B') \mid Y\big]\big\}\,.$$

(c) (9 points) Derive the Euler equation for bond issuance using the envelope theorem. Interpret the equation briefly (you may assume the value function is differentiable).

Solution: Apply the envelope theorem to $V_{nd}(Y, B)$ with respect to B, holding B' at its optimum:

$$\frac{\partial V_{nd}}{\partial B}(Y, B) = -u'(C),$$

since $\partial C/\partial B = -1$. In states where the country repays, $V_B(Y, B) = -u'(C)$; in states where it defaults, $V_B(Y, B) = 0$ (since V_d does not depend on B).

The first-order condition for B' in V_{nd} is

$$\big[q(Y, B') + B' q_{B'}(Y, B')\big]u'(C) + \beta\, \mathbb{E}\big[V_B(Y', B') \mid Y\big] = 0.$$

Substituting the envelope result,

$$\big[q(Y, B') + B' q_{B'}(Y, B')\big]u'(C) = \beta\, \mathbb{E}\big[u'(C')\, \mathbf{1}\big\{V_{nd}(Y', B') \geq V_d(Y')\big\} \,\big|\, Y\big]\,.$$

The left-hand side is the marginal utility gain from issuing one extra unit of debt today: the country receives price q, but issuing more also depresses the price on all newly issued bonds through $q_{B'}$, so the net marginal revenue is $q + B' q_{B'}$. The right-hand side is the discounted expected marginal utility cost of repaying tomorrow, but only in states where the country does not default — the indicator $\mathbf{1}\{\cdot\}$ reflects the option to default.