PhD in Business Economics - Insper
Macroeconomics 1
Professor: Tomás Martinez
Final 2025

Instructions:

- Write your name on the first page of the exam.
- The exam consists of 3 questions totaling 105 points (5 extra points).
- The duration of the exam is 3:00 hours.
- It is allowed to consult any material. It is not allowed to communicate with other individuals.
- Keep the exam organized and the pages numbered.
- If something in the question is unclear, write the assumptions you deem necessary to have a well-defined problem and proceed.
- If you get stuck on a specific part of a question, remember that you may consider the result of that part as given and continue to answer the other parts.

Questions

1. **(RBC with two agents – 31 points).** Consider the RBC model with two types of agents: "Ricardian" agents and "Keynesian' agents. There is a unit mass of agents in the economy, where a fraction γ of the agents are Ricardian.

   Keynesian agents are *hand-to-mouth*, meaning they do not have access to capital markets and therefore do not save. Ricardian agents are the typical agents considered in the standard model. They choose how much to save and therefore accumulate capital. Both types of agents choose how much to consume and how much to work.

   Agents have the same utility function. Let $i \in R$, K denote the type of agent, where R indicates Ricardian and K indicates Keynesian. The utility function is:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t^i + \theta \log(1 - N_t^i) \right\},$$

   where C_t^i and N_t^i denote consumption and labor, and $\beta \in (0, 1)$.

   The Ricardian agent's budget constraint is:

$$C_t^R + K_{t+1}^R = (1 - \tau_n) W_t N_t^R + (1 + r_t - \delta) K_t^R \qquad \forall t.$$

   The Keynesian agent's budget constraint is:

$$C_t^K = (1 - \tau_n) W_t N_t^K \qquad \forall t,$$

   where τ_n is the tax on labor earnings (tax revenues are thrown into the ocean). The aggregate production function is: $Y_t = Z_t K_t^\alpha N_t^{1-\alpha}$, where $\alpha \in (0, 1)$. The shock follows an AR(1) stochastic process: $\log Z_t = \rho \log Z_{t-1} + \sigma \varepsilon_t$, where ε_t has mean 0 and standard deviation 1. There is no population growth. $K_0 > 0$ is given. Assume the transversality condition is satisfied.

   (a) (10 points) Describe and solve the problem of the Ricardian agent in a competitive environment. Derive the Euler equation and the labor supply equation. Which condition is intertemporal and which is intratemporal? Does the labor tax distort the hours decision of the Ricardian agents?

   Solution: Ricardian agents are the standard RBC agents, where consumption depends on permanent income. The problem is standard:

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t (\log(C_t^R) + \theta \log(1 - N_t^R)) + \lambda_t ((1 - \tau_n) W_t N_t^R + (1 + r_t - \delta) K_t^R - C_t^R - K_{t+1}^R)$$

   Solving the first-order conditions, we obtain the Euler equation and the labor supply equation:

$$(EE) \qquad \frac{1}{C_t^R} = \mathbb{E}_t\left[\beta(1 + r_{t+1} - \delta)\frac{1}{C_{t+1}^R}\right] \qquad \forall t$$

$$(LS) \qquad \frac{\theta}{1 - N_t^R} = \frac{W_t(1 - \tau_n)}{C_t^R} \qquad \forall t$$

The Euler equation is the intertemporal condition, and the labor supply equation (LS) is the intratemporal condition.

(b) (13 points) Describe and solve the problem of the Keynesian agent. Derive the Frisch elasticity for the Keynesian agent. Write optimal consumption and hours worked as functions of prices and parameters. Does the labor tax distort the hours decision of the Keynesian agents?

Solution: For Keynesian agents, the problem is static. For each t, the problem reduces to:

$$\max_{C_t^K,\ N_t^K} \log(C_t^K) + \theta \log(1 - N_t^K) \qquad \text{s.t.} \quad C_t^K = W_t N_t^K (1 - \tau_n)$$

First-order conditions imply the same labor supply equation as for the Ricardian agent:

$$\frac{\theta}{1 - N_t^K} = \frac{W_t(1 - \tau_n)}{C_t^K} \qquad \forall t$$

The Frisch elasticity is defined as $\frac{d \log N_t^K}{d \log W_t}$ holding C_t^K constant. Following standard derivation:

$$\frac{d \log N_t^K}{d \log W_t} = \left(\frac{1 - N_t}{N_t}\right).$$

To solve for N_t^K and C_t^K as functions of parameters:

$$\frac{\theta}{1 - N_t^K} = \frac{1}{N_t^K} \Leftrightarrow N^K = \frac{1}{1 + \theta} \qquad\qquad C_t^K = \frac{W_t(1 - \tau_n)}{1 + \theta}.$$

Thus, the Frisch elasticity is the same for both types of agents, but hours worked do not fluctuate for Keynesian agents. This happens because they do not save, so any change in wages directly translates into consumption, and the wealth effect completely cancels out the substitution effect in the labor supply equation.

(c) (8 points) Suppose prices are given by $r_t = Z_t\alpha(K_t/N_t)^{\alpha-1}$ and $W_t = Z_t(1-\alpha)(K_t/N_t)^{\alpha}$. How does the steady-state capital-labor ratio, $\bar{K}/\bar{N}$, change when the fraction of Ricardian agents in the economy increases ($\uparrow \gamma$)?

Solution: It does not change. Note that from the Ricardian Euler equation:

$$\frac{1}{C_t^R} = \mathbb{E}_t\left[\beta(1 + Z_{t+1}\alpha(K_{t+1}/N_{t+1})^{\alpha-1} - \delta)\frac{1}{C_{t+1}^R}\right],$$

we obtain the steady-state capital-labor ratio, $\bar{K}/\bar{N}$, as:

$$\beta(1 + \bar{Z}\alpha(\bar{K}/\bar{N})^{\alpha-1} - \delta) = 1,$$

which does not depend on γ.

2. (***Herdeiros* and inheritance tax - 40 points).** Consider an OLG model where a continuum of individuals with unit mass live for two periods: childhood and adulthood. Individuals have warm-glow preferences, meaning they derive utility from the bequests they leave to their children (*os herdeiros*). In adulthood, each individual receives an inheritance from their parents (which they rent out as capital to firms), has children, works, chooses the inheritance to leave for their own children, and then dies. Utility during childhood is not relevant, while the utility of an adult at time t is given by:

$$u(c_t, b_t) = \ln(c_t) + \beta \ln(b_t),$$

where b_t is the bequest left to their children and $\beta \in (0,1)$. Bequests are subject to an inheritance tax, τ_i, and all tax revenue is "thrown into the ocean". The budget constraint is:

$$c_t + b_t = w_t + (1 + r_t)b_{t-1}(1 - \tau_i),$$

where $b_0 > 0$ is given. For simplicity, assume there is no depreciation ($\delta = 0$), and the production function is Cobb-Douglas: $Y_t = K_t^\alpha L_t^{1-\alpha}$, with $\alpha \in (0,1)$. There is no population growth, and $L_0 = 1$.

(a) (10 points) Solve the household's problem. Find the optimal consumption and bequest levels as functions of prices, parameters, taxes, and the bequests received.

Solution: The first-order conditions are:

$$\frac{1}{c_t} = \lambda_t \qquad \& \qquad \frac{\beta}{b_t} = \lambda_t \quad \forall t.$$

Using the FOCs and the budget constraint, we find:

$$\begin{aligned} c_t &= \frac{1}{1+\beta}[w_t + (1 + r_t)b_{t-1}(1 - \tau_i)] \quad \forall t, \\ b_t &= \frac{\beta}{1+\beta}[w_t + (1 + r_t)b_{t-1}(1 - \tau_i)] \quad \forall t. \end{aligned}$$

(b) (10 points) Write the asset market equilibrium condition for this economy. Derive a difference equation that characterizes the evolution of capital per capita, k_t, over time. You can assume that $r_t = \alpha k_t^{\alpha-1}$ and $w_t = (1-\alpha)k_t^\alpha$.

Solution: The asset market equilibrium condition is $k_{t+1} = b_t$. Substituting b_t, r_t, and w_t in the equation above:

$$\begin{aligned} k_{t+1} &= \frac{\beta}{1+\beta}[(1-\alpha)k_t^\alpha + (1 + \alpha k_t^{\alpha-1})k_t(1 - \tau_i)] \\ k_{t+1} &= \frac{\beta}{1+\beta}[(1 - \alpha + \alpha(1 - \tau_i))k_t^\alpha + k_t(1 - \tau_i)]. \end{aligned}$$

(c) (8 points) Find the steady state value of k_{ss}. How does the steady-state capital stock change in response to a change in the inheritance tax τ_i?

Solution: Setting $k_{ss} = k_{t+1} = k_t$ and using the equation above:

$$\begin{aligned} k_{ss} &= \frac{\beta}{1+\beta}[(1-\alpha+\alpha(1-\tau_i))k_{ss}^{\alpha} + k_{ss}(1-\tau_i)] \\ \Leftrightarrow \quad k_{ss} &= \left[\frac{\beta(1-\alpha+\alpha(1-\tau_i))}{1+\beta-\beta(1-\tau_i)}\right]^{\frac{1}{1-\alpha}}. \end{aligned}$$

We can see that an increase in τ_i reduces the numerator and increases the denominator, leading to a lower steady-state capital stock in the economy.[1]

(d) (12 points) Now suppose that the *herdeiros* decide how much to work in the labor market. The utility function is now:

$$u(c_t, b_t, n_t) = \ln(c_t) + \beta \ln(b_t) - \theta n_t,$$

and the budget constraint becomes:

$$c_t + b_t = n_t w_t + (1+r_t)b_{t-1}(1-\tau_i).$$

Solve for the agents' optimal labor supply. How does the inheritance tax affect their labor supply decision? (*Hint*: Note that the FOCs for c_t and b_t remain unchanged.)

Solution: The first-order condition of n_t implies:

$$\theta = w_t \lambda_t \qquad \Leftrightarrow \qquad c_t = \frac{w_t}{\theta}.$$

Substituting in the budget constraint and solving for n_t, we get:

$$\begin{aligned} (1+\beta)c_t &= n_t w_t + (1+r_t)b_{t-1}(1-\tau_i) \\ (1+\beta)\frac{w_t}{\theta} &= n_t w_t + (1+r_t)b_{t-1}(1-\tau_i) \\ \Leftrightarrow \qquad n_t &= \frac{1+\beta}{\theta} - \frac{(1+r_t)b_{t-1}(1-\tau_i)}{w_t}. \end{aligned}$$

Thus, an increase in τ_i creates an income effect on the *herdeiros*, leading them to work more.

[1] For a more complete analysis of inheritance tax see Piketty and Saez, ECTA 2013.

3. **(Idiosyncratic Shocks and Endogenous Separations in the DMP Model - 34 points).** Consider the standard Search and Matching framework: there is a unit measure of individuals, each of whom is either employed or unemployed. An individual i discounts the future at rate β and has linear utility over the consumption good, $c_{i,t}$. An employed worker receives the wage w_t, while an unemployed worker receives b. Wages are determined through Nash bargaining. When a vacancy is open, the firm incurs a flow cost of κ. The job-finding and vacancy-filling probabilities are given by $\lambda_w(\theta_t)$ and $\lambda_f(\theta_t)$, respectively, where labor market tightness is defined as $\theta_t \equiv v_t/u_t$.

   The key difference in this model is that job productivity has two components: a common component across all jobs, z, and a job-specific component, x, where x is drawn from a cumulative distribution function $G(x)$ with support $[\underline{x}, \overline{x}]$. A matched firm produces zx units of output, and all matches start at the maximum productivity level, $\overline{x}$. For simplicity, assume that z is constant.

   Separation occurs *endogenously*. With probability μ, the firm draws a new job-specific productivity component x in the next period; otherwise, the productivity remains the same. If the new x falls below a reservation productivity threshold x^R, the match dissolves.[2] There is no exogenous separation.

   (a) (10 points) Write the law of motion for unemployment in this economy.

   Solution: You need to adjust the job destruction probability; the rest is standard. The law of motion for unemployment in this economy is:

$$u_{t+1} = u_t(1 - \lambda_w(\theta_t)) + (1 - u_t)\mu G(x^R)$$

   (b) (8 points) Write the Beveridge curve. What happens to it if there is an increase in the reservation productivity x^R? *Hint:* Think about the changes of the separation rate in this economy.[3]

   Solution: The Beveridge curve is determined by the steady state of the unemployment law of motion:

$$u_{ss} = \frac{\mu G(x^R)}{\mu G(x^R) + \lambda_w(v_{ss}/u_{ss})}$$

   Note that the reservation productivity x^R is a sufficient statistic for the separation rate: an increase in x^R raises $G(x^R)$, thereby increasing the separation rate. A higher separation rate increases steady-state unemployment for any given vacancy rate, shifting the Beveridge curve to the right.

   (c) (10 points) Write the Bellman equation for a matched firm, J, and for an open vacancy, V. What are the state variables in these value functions?

   Solution: Since we are assuming that z is constant, the only state variable for the matched firm is x. The value of an open vacancy is constant. If z were not constant, it would also be a state variable.

   The Bellman Equations are:

[2] Assume the parameters are such that $x^R \in (\underline{x}, \overline{x})$.

[3] Note that x^R is an endogenous variable, but for the purpose of this question you can treat it as exogenous.

$$J(x) = zx - w(x) + \beta[(1-\mu)J(x) + \mu \int_{x^R}^{\overline{x}} J(x')dG(x') + \mu \int_{\underline{x}}^{x^R} V dG(x')],$$

$$\Leftrightarrow \quad J(x) = zx - w(x) + \beta[(1-\mu)J(x) + \mu \int_{x^R}^{\overline{x}} J(x')dG(x') + \mu V dG(x^R)],$$

and the value of a firm with open vacancy is

$$V = -\kappa + \beta[\lambda_f(\theta)J(\overline{x}) + (1 - \lambda_f(\theta))V].$$

(d) (6 points) Write an equation that implicitly pins down the reservation productivity x^R that includes J, w and parameters (you do not need to solve it). You can assume that free entry holds.

Solution: The equation that pins down is given by $J(x^R) = V = 0$, where the last equality comes from the free entry condition. Using the Bellman Equation above:

$$J(x^R) = zx^R - w(x^R) + \beta[(1-\mu)J(x^R) + \mu \int_{x^R}^{\overline{x}} J(x')dG(x')] = 0$$

$$zx^R - w(x^R) + \beta\mu \int_{x^R}^{\overline{x}} J(x')dG(x') = 0,$$

where in the first equation we use the free entry condition, and in the second, we use the fact that $J(x^R) = 0$.[4]

[4] You could also derive this from the worker's Bellman equation by using the condition $W(x^R) = U$. See Pissarides (2000), Chapter 2, for a discussion of this model.