PhD in Business Economics - Insper
Macroeconomics 1
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Instructions:

- Write your name on the first page of the exam.
- The exam consists of 3 questions totaling 100 points.
- The duration of the exam is 2:30 hours.
- It is allowed to consult any material. It is not allowed to communicate with other individuals.
- Keep the exam organized and the pages numbered.
- If something in the question is unclear, write the assumptions you deem necessary to have a well-defined problem and proceed.
- If you get stuck on a specific part of a question, remember that you may consider the result of that part as given and continue to answer the other parts.

Questions

1. **(Progressive Taxation and the Laffer Curve - 33 points).** Consider the following neoclassical growth model with elastic labor supply. There is a representative household with a mass of one, exhibiting GHH utility function:[1]

$$\sum_{t=0}^{\infty} \beta^t U\left(c_t - \frac{n_t^{1+\phi}}{1+\phi}\right),$$

where $U()$ satisfies the usual properties: $U'() > 0$, $U''() < 0$ and the Inada conditions. The household budget constraint is given by

$$c_t + a_{t+1} = \text{ disposable labor income } + (1+r_t)a_t \qquad \forall t.$$

The disposable (after-tax) labor income is given by a log-linear function:

$$\text{disposable labor income } \equiv w_t n_t - T(w_t n_t) = \lambda (w_t n_t)^{1-\tau},$$

where the parameters λ and τ control the level and the progressivity of the tax system. As $\tau \to 1$, the system becomes more progressive, while a lower λ implies a higher average tax rate.

The function $T(w_t n_t)$ represents the aggregate tax revenue, which is used to finance government expenditure, G_t. The government balance its budget every period, i.e., it does not carry debt.

There is no capital, and the production of the final good requires only labor: $y_t = z_t n_t$. There is no population growth or technological change.

(a) (9 points) Solve the household problem in this economy. Find an intertemporal and an intratemporal condition that gives the optimal household decisions.

Solution: After the usual steps, one can find the Euler Equation and the Labor Supply Equation.

$$(EE) \qquad U'\left(c_t - \frac{n_t^{1+\phi}}{1+\phi}\right) = \beta(1+r_{t+1})U'\left(c_{t+1} - \frac{n_{t+1}^{1+\phi}}{1+\phi}\right) \qquad \forall t$$

$$(LS) \qquad n_t = [(1-\tau)\lambda w_t^{1-\tau}]^{\frac{1}{\phi+\tau}} \qquad \forall t$$

(b) (6 points) Write the equilibrium conditions for this economy.

Solution: The equilibrium conditions of this economy for all periods:

Goods Market	$y_t = c_t + G_t$
Asset Market	$a_t = 0$
Labor Market	$n_t^d = n_t^s$

[1] GHH preferences denote *Greenwood, Hercowitz, and Hoffman.*

(c) (9 points) Find an equation that determines the number of hours worked n_t as a function of parameters and exogenous variables. What happens to the hours worked when the government changes the parameter that governs the average tax rate, λ? How does this effect depend on the degree of progressivity of the tax system?

Solution: First, notice that the equilibrium wage is given by the marginal product of labor: $w_t = z_t$. Then, the equation that determines the hours worked in the economy is:

$$n_t = [(1-\tau)\lambda z_t^{1-\tau}]^{\frac{1}{\phi+\tau}}$$

Taking logs and the derivative w.r.t. to $\ln \lambda$, we have:

$$\frac{\partial \ln n_t}{\partial \ln \lambda} = \frac{1}{\phi + \tau}.$$

Thus, an increase in λ (or a decrease in the tax level of the economy) increases the hours worked. This effect is stronger the more progressive the tax system. This happens because workers internalize that if they increase hours, they will face a higher marginal tax rate.

(d) (9 points) Suppose the government wants to find the value of λ that maximizes aggregate tax revenue. Formulate a problem that allows the government to determine the optimal λ (you do not need to solve it). Interpret the problem and discuss the trade-offs the government faces.

Solution: The aggregate tax revenue is given by the following expression:

$$T(w_t n_t) = w_t n_t - \lambda (w_t n_t)^{1-\tau}.$$

Using the solution to the previous questions, we can derive the Laffer Curve as a function of parameters:

$$\begin{aligned} \max_{\lambda} \quad & zn - \lambda (zn)^{1-\tau} \\ \text{s.t.} \quad & n = [(1-\tau)\lambda z^{1-\tau}]^{\frac{1}{\phi+\tau}} \end{aligned}$$

The trade-off is the following:

- Higher λ (i.e., lower tax) incentivizes people to work more hours, increasing the tax base;
- Higher λ (i.e., lower tax) decreases the tax rate, decreasing tax revenues;

2. **(Bitcoin in the OLG Model - 33 points).** Consider the two period OLG model, where agents work when young and consume their savings when old. The population grows at rate n, with $L_0 = 1$. The utility function of a generation born in period t is:

$$\ln(c_t^1) + \beta \ln(c_{t+1}^2).$$

Agents can save in the form of productive capital, s_t, at a return rate r_t equal to the marginal product of capital, or by buying Bitcoin, b_t, at price p_t. Bitcoin is useless and does not yield dividends, but the agent can sell it when old (if anyone is willing to buy it) at price p_{t+1}. The budget constraint when the agent is young and old are given by:

$$c_t^1 + s_t + p_t b_t \leq w_t$$
$$c_{t+1}^2 \leq (1 + r_{t+1})s_t + p_{t+1}b_t$$

The supply of Bitcoin is constant and is given by $M > 0$, so the aggregate value of it is given by $b_t L_t \equiv B_t = M p_t$.

The initial old generation starts with $K_0 > 0$ given. The aggregate production function is $Y_t = K_t^\alpha L_t^{1-\alpha}$ with $0 < \alpha < 1$ and the markets are competitive.

(a) (9 points) Solve the household problem in this economy. Find a equation that shows the trade-off between consumption and savings and a *no-arbitrage* equation that relates the gross return of Bitcoin, p_{t+1}/p_t, to the rate of return of capital.

Solution: A simple Lagragian yields the solution. The two equations are the Euler equation (EE) and the no-arbitrage equation (NA):

$$\begin{aligned} &(EE) \qquad c_{t+1}^2 = c_t^1 \beta(1 + r_{t+1}) \qquad \forall t, \\ &(NA) \qquad \frac{p_{t+1}}{p_t} = (1 + r_{t+1}) \qquad \forall t \end{aligned}$$

(b) (8 points) Use the no-arbitrage equation and the marginal product of capital to find a first-order difference equation of b_{t+1} as a function of b_t and k_{t+1}.

Solution:

$$\begin{aligned} p_{t+1} =& p_t(1 + r_{t+1}) \\ B_{t+1} =& B_t(1 + f'(k_{t+1})) \\ b_{t+1} =& \frac{b_t(1 + f'(k_{t+1}))}{1 + n}. \end{aligned}$$

(c) (7 points) Write the asset market equilibrium condition. Use this equation to find a first-order difference equation on capital.

Solution: The asset market eq. condition must consider the bubbly asset:

$$
\begin{aligned}
K_{t+1} =& L_t s(w_t, r_{t+1}) - B_t \\
k_{t+1} =& \frac{s(w_t, r_{t+1}) - b_t}{1+n} \\
k_{t+1} =& \frac{s(f(k_t) - k_t f'(k_t), f'(k_{t+1})) - b_t}{1+n} \\
k_{t+1} =& \frac{s(k_t, k_{t+1}) - b_t}{1+n}.
\end{aligned}
$$

(d) (9 points) Suppose the economy is dynamically efficient in the steady state: $f'(k_{ss}) - n > 0$. Argue whether individuals in this economy would or would not buy Bitcoin in the steady state (*Hint:* think about how to find capital in the steady state).

Solution: The steady state in this model is defined by these two equations:

$$
\begin{aligned}
b_{ss} =& \frac{b_{ss}(1 + f'(k_{ss}))}{1+n}, \\
k_{ss} =& \frac{s(k_{ss}, k_{ss}) - b_{ss}}{1+n}.
\end{aligned}
$$

By manipulating the first equation, one could easily see that if $b_{ss} > 0$, it must be that $f'(k_{ss}) = n$. If $f'(k_{ss}) \neq n$, it must be $b_{ss} = 0$ so the first equation is satisfied. Therefore, in the dynamically efficient steady state, the individuals WILL NOT buy Bitcoin.

Another way to see this is by looking at the no-arbitrage condition. In the dynamically efficient case, the interest rate is higher than the population growth rate, $r > n$. By the no-arbitrage condition, this implies that $p_{t+1} > p_t$, and the aggregate value of Bitcoin would explode, eventually becoming larger than the income of the young.

In the dynamically inefficient case, $f'(k_{ss}) - n \leq 0$, there will be two equilibria: one where Bitcoin is valued in the steady state ($f'(k_{ss}) = n$) and the other where it is not ($f'(k_{ss}) < n$). Which equilibrium the economy will end up in depends on the initial conditions (for a full discussion, see Ch. 5 of Blanchard and Fischer, Lectures on Macroeconomics, 1989).

3. **(Search Effort - 34 points).** Consider a problem where a measure of one of individuals is either employed or unemployed. An individual i discounts the future by β and has linear utility over the consumption good, $c_{i,t}$. A employed worker receivers the wage, w_t, while a unemployed worker receives b.

   An unemployed worker looks for a job by exerting variable search effort, $s_{i,t} \geq 0$. The cost of searching for a job depends on how intensively the worker searches and is given by a function $h(s)$, where $h'(s) > 0$, $h''(s) > 0$ and $h'(0) = 0$. The utility function of a worker is given by:

$$\sum_{t=0}^{\infty} \beta^t [c_t - h(s_t)].$$

   Let $S_t \equiv \int_0^1 s_i d_i$ denote the total search intensity exerted by the individuals. The matching function is given by $M = S^\eta v^{1-\eta}$, so

$$p(\theta) \equiv \frac{M}{S}, \qquad \text{where the tightness is } \theta \equiv \frac{v}{S}.$$

   The individual worker's job-finding probability is given by $s_i p(\theta_t)$. Employed workers separate at a rate $\sigma \in (0,1)$.

   (a) (8 points) State the Bellman equations for an employed and an unemployed individual.
   Solution:

$$\begin{aligned} \text{Employed} \qquad W_t &= w_t + \beta((1-\sigma)W_{t+1} + \sigma U_{t+1}) \\ \text{Unemployed} \qquad U_t &= \max_{s \geq 0}\{b - h(s) + \beta(sp(\theta_t)W_{t+1} + (1 - sp(\theta))U_{t+1})\} \end{aligned}$$

   (b) (7 points) Find an equation that determines the search effort exerted by a worker.
   Solution: Taking the f.o.c with respect to s:

$$h'(s_t) = \beta p(\theta_t)(W_{t+1} - U_{t+1}),$$

   where the LHS is the marginal cost of search and RHS is the marginal benefit of putting additional effort into searching for a job

   (c) (9 points) State the law of motion of unemployment. Find an equation that describes the Beveridge Curve in this model.

   Solution: Note that all unemployed workers are equal, hence the optimal search effort is $s_{i,t} = s_t$. Moreover, since the search effort of employed agents is 0, the aggregate search effort is $S_t = s_t u_t$.
   Thus, the law of motion of unemployment is

$$\begin{aligned} u_{t+1} &= u_t + u_t s_t p(\theta_t) - \sigma(1 - u_t) \\ u_{t+1} &= u_t + u_t s_t \frac{M_t}{s_t u_t} - \sigma(1 - u_t) \\ u_{t+1} &= u_t + (u_t s_t)^\eta v_t^{1-\eta} - \sigma(1 - u_t) \end{aligned}$$

   In the steady state:

$$(u_{ss}s_{ss})^{\eta}v_{ss}^{1-\eta} = \sigma(1-u_{ss}),$$

and we get the Equation that describes the Beveridge curve.

(d) (10 points) Suppose that the firm block of the model is standard and the job creation equation is the same as the one seen in class (in the steady state):

$$\beta(z-w) = \frac{\kappa}{q(\theta_{ss})}(1-\beta(1-\sigma))$$

where q is the vacancy-filling probability, and z is the productivity of the firm. For simplicity, assume that wages are fixed, $w \in (z, b)$. Suppose there is a permanent increase in the productivity of firms $z' > z$. How does the steady-state equilibrium of vacancies and unemployment change? Describe how search effort impacts these variables relative to the baseline without search effort.

Solution: Relative to the baseline model, the model with search effort has one additional endogenous variable, s, and one additional equation (the equation from part (b)). From the Equation in (b), an increase in z has two effects:

- The increase in z incentivize vacancy creation: $\uparrow \theta \Rightarrow \uparrow p(\theta) \rightarrow \uparrow s$.
- The increase in search effort increases the cost of search $h(s)$, which decreases the value of being unemployed, U. This amplifies the effort even further.

In the model without search effort, the job creation curve becomes more inclined after an increase in z. Relative to the model without search effort, the job creation curve has an extra term (in red):

$$\beta(z-w) = \frac{\kappa}{(v/u)^{-\eta}s^{\eta}}(1-\beta(1-\sigma)).$$

Since, s increases after an increase in z, the job creation becomes even more inclined compared to the case without search effort.

Finally, without search effort, the Beveridge curve does not change after a change in z. In a model with search effort, the Beveridge curve shifts to the left after an increase in z (see the equation in part (c)). Thus, relative to the baseline model, adding search effort generates more amplification. For a full analysis, see Gomme and Lkhagvasuren (2015, JME).