International Economics II

Theoretical Analysis of CA Determination: Production Economies

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Outline

1. The Current Account in a Production Economy
2. The Savings Schedule
3. Applications and Extensions

Motivation

- We now have a model of the current account, which relates its fluctuations to changes in the domestic economy:
  - Output (national income): endowment
  - Private consumption: micro-founded
  - Investment: not yet
  - Government spending: not yet
- Investment, which consists of spending in capital goods such as machines, new structures, equipment, and inventories, is an important component of aggregate demand amounting to around 20 percent of GDP in most countries.
- Further, investment is the most volatile component of aggregate demand, and, as such, it is important for understanding movements in the current account over the business cycle.

Motivation

- We will add investment in physical capital to our theoretical model of current account determination.
- To better understand it, as well as its price, we will next enrich the framework with some simple microfoundations of firms' behavior.
- Then, we will study the adjustment process of the economy to a variety type of economic shocks.

Outline

1. The Current Account in a Production Economy

2. The Savings Schedule

3. Applications and Extensions

A Production Economy

- Two-period economy: Output is produced by homogeneous firms using only physical capital.

- Output in period $t+1$, Q_{t+1} is

$$Q_{t+1} = A_{t+1} F(I_t). \tag{1}$$

  - $A_{t+1} > 0$ is the level of technology at $t+1$;
  - $F(.)$ s the production function: $F'(.) > 0$, $F''(.) < 0$, and $F(0) = 0$;
  - I_t denotes investment in physical capital at t which is transformed into output the following period.

- Investment decision of period t determines the capital stock in period $t+1$.

- By including gross investment in the production function, we are implicitly assuming that (a) all capital in a given period is used for production, and (ii) the depreciation rate is zero.

Marginal Product of Capital

- Consider firm's production in period 2, $Q_2 = A_2F(I_1)$. Then, the marginal returns to capital, MPK, in period 2 is

$$MPK = A_2F'(I_1) > 0, \tag{2}$$

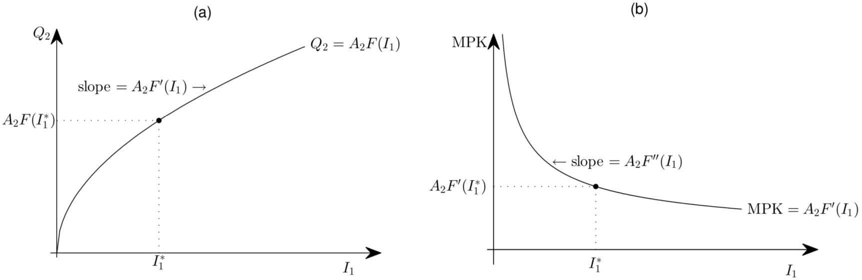
which is always positive given the assumption made on the production function.

- Further, the MPK is decreasing with further investment:

$$MPK' = A_2F''(I_1) < 0. \tag{3}$$

- This property is known as diminishing marginal product of capital.
- A simple example that satisfies this properties: $F(I_t) = I_t^{\alpha}$ where $0 < \alpha < 1$.

The Production Function



Production Decisions of the Firm

- We will assume that firms have no retained earnings and therefore need to borrow in order to finance the purchase investment goods. In period 1:

$$D_1^f = I_1, \tag{4}$$

  where D_1^f is the amount of debt issued by firms at the interest rate r_1.

- In period 2, Firms produce and then sell the output: $A_2F(I_1)$.

- The also have to pay back the loan: $(1 + r_1)D_1^f$

Profit Maximization

- Given that the only input of production is capital, we can then write period-2 profits as

$$\Pi_2 = A_2 F(I_1) - (1 + r_1) D_1^f \tag{5}$$

- Substituting $I_1 = D_1^f$, we can write the firm's maximization problem:

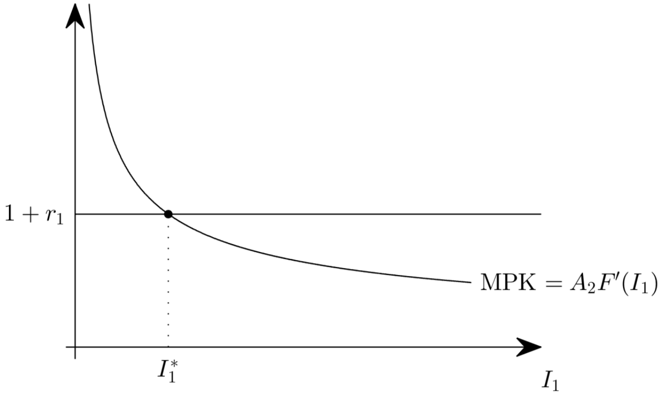
$$\max_{I_1} \Pi_2 = A_2 F(I_1) - (1 + r_1) I_1 \tag{6}$$

- Equalizing the FOC to zero yields the solution:

$$A_2 F'(I_1) = 1 + r_1 \tag{7}$$

- Hence, the Marginal Cost of borrow more at t (the interest rate $1 + r_1$) is equal to the Marginal Benefit of producing an additional unit at $t + 1$ (MPK).

Marginal Product of Capital and Marginal Cost Schedules



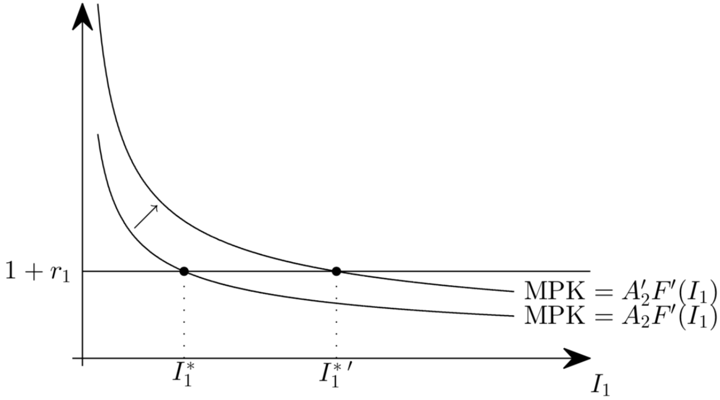
Tracing out the Investment Schedule

- What happen to investment I_1^* when the interest rate or technology changes? What about profits Π_2^*?
- We know the optimal investment is a function of r_1 and A_2:

$$
\begin{aligned}
A_2 F'(I_1^*) &= 1 + r_1 \\
F'(I_1^*) &= \frac{1 + r_1}{A_2} \\
I_1^* &= F'^{-1}\left(\frac{1 + r_1}{A_2}\right) = I(r_1; A_2) \qquad (8)
\end{aligned}
$$

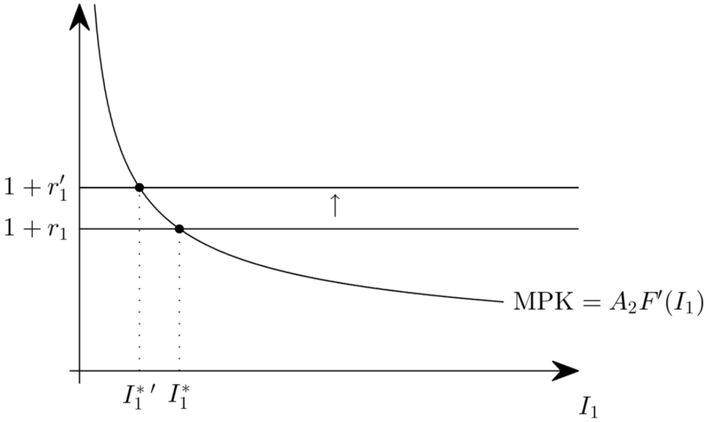
  where $F'^{-1}(.)$ is the inverse of $F'(.)$.
- Let's take a look how graphically.

Increase in Technology



- Increase in $A \Rightarrow$ increase in $MPK \Rightarrow$ rise in firm's demand for capital.
- Profits are equal to the area below MPK and above $MC = (1 + r_1) \Rightarrow$ increase.

Increase in Interest Rate



- Increase in $r \Rightarrow$ increase in $MC \Rightarrow$ fall in firm's demand for capital.
- Profits are equal to the area below MPK and above $MC = (1 + r_1) \Rightarrow$ fall.

Tracing out the Investment Schedule

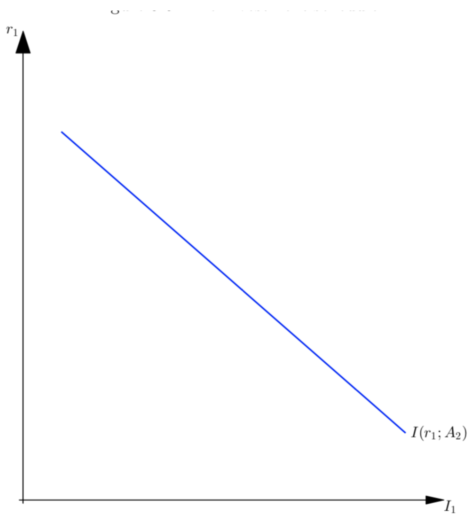
- Hence, we can summarize investment and profits as a function of r and A:

$$I_1 = I(\underset{-}{r_1}; \underset{+}{A_2}) \tag{9}$$

$$\Pi_2 = \Pi(\underset{-}{r_1}; \underset{+}{A_2}) \tag{10}$$

- We will refer to (9) as the investment schedule. Though written for period 1, such an equation will hold at any period $t \geq 1$.
- Remember that all firms are identical, hence aggregate investment is the sum of the investment decisions of individual firms and it behaves just like investment at firm level.
- We can depict the investment schedule in the following figure, where we relate investment and the interest rate for a given level of technology.

Investment Schedule



What happens for an exogenous change in r? and in A?

Closing the (Intertemporal) Firm's Problem

- We now have an investment schedule that depicts the firm's investment decision for time period 1 (and greater in multiple period models).
- But, what about production at $t = 1$, and thus profits at $t = 1$?:

$$\Pi_1 = A_1 F(I_0) - (1 + r_0) D_0^f \tag{11}$$

  with $D_0^f = I_0$.

- As is standard intertemporal models, we will take period 0 variables as predetermined, the firm takes r_0, I_0, and D_0^f as exogenous.
- Furthermore, since A_1 is out of the control of the firm, profits at $t = 1$ are not affected by any decisions made by the firm.

The Consumption-Saving Decision of Households

- As before, the household will maximize lifetime utility subject to her intertemporal budget constraint.
- We will follow a similar logic, but with some wrinkles given that we now have production and firms in the model.
- In particular, we will allow households to consume from:
  1. Borrowing (lending if a saver)
  2. Distribution of firm profits. I.e., the household owns the firm.
- Hence, instead of receiving an endowment each period, households receive profit payments from firms, $\Pi_1(r_0, A_1)$ in period 1 and $\Pi_2(r_1, A_2)$ in period 2.

Intertemporal Budget Constraint

- The household is endowed with a B_0^h units of bonds, which pays an interest rate r_0 on bonds held between 0 and 1, and owns the rights to the firm's period 1 profits.

- The period 1 and 2 budget constraints are:

$$C_1 + B_1^h - B_0^h = \Pi_1(r_0, A_1) + r_0 B_0^h \tag{12}$$

$$C_2 + B_2^h - B_1^h = \Pi_2(r_1, A_2) + r_1 B_1^h \tag{13}$$

- Using the usual transversality condition $B_2^h = 0$ and substituting B_1^h, we get:

$$C_1 + \frac{C_2}{1+r_1} = \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1+r_2} + (1+r_0)B_0^h \tag{14}$$

- This intertemporal budget constraint looks similar to what we derived for the endownment economy, except now we replace endowments (Q_t) with profits (Π_t).

Utility Maximization

- The household chooses C_1 and C_2 to maximize its utility function $U(C_1, C_2)$ subject to the intertemporal budget constraint:

$$C_2 = (1 + r_1)(\overline{W} - C_1) \tag{15}$$

where:

$$\overline{W} = \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_2} + (1 + r_0)B_0^h \tag{16}$$

- The household takes $\overline{W}$ as exogenously given.

- The first-order conditions imply that the indifference curve is tangent to the intertemporal budget constraint, with a slope $-(1 + r_1)$, so optimal consumption must satisfy:

$$\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = 1 + r_1 \tag{17}$$

- Nothing new here, these optimality conditions are identical to those pertaining to the endowment economy.

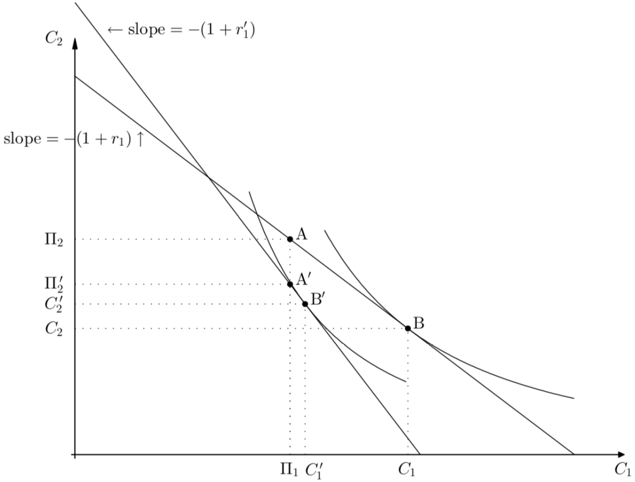
Effects of Shocks on Consumption

- Changes in A_1 and A_2 will work exactly as an endowment shock: $\uparrow A_1 \Rightarrow \uparrow \Pi_1$ and $\uparrow A_2 \Rightarrow \uparrow \Pi_2$.
- If the change in A_1 is transitory, the household will borrow to finance its consumption.
- If the change in A_1 is permanent, the household will adjust its consumption.
- However, an increase in r_1 will have an additional effect by reducing Π_2.

Effects of r_1 on Consumption

- An increase in r_1 will have two income effects and one substitution effect:
  1. Substitution Effect: The increase in r_1 induces households to substitute present consumption with future consumption. Same as in the endowment economy.
  2. Income Effect 1: The increase in r_1 makes debtors poorer and creditors richer. Same as in the endowment economy.
  3. Income Effect 2: The increase in r_1 will reduce Π_2, making the household poorer. Absent in the endowment economy!
- We usually assume that the substitution effect always dominates the income effect 1 (case with log utility) $\Rightarrow$ fall in C_1.
- The income effect 2 is clearly negative and further reduces C_1.

Effects of r_1 on Consumption



Equilibrium

- We have characterized the optimal behavior of firms and households individually. We can put their optimal conditions and constraints together to solve for equilibrium.
- To begin, remember that this is a small open economy, so the interest rate must equal the world rate in equilibrium:

$$r_1 = r^* \tag{18}$$

- Next, we can solve for the economy's initial NFA position, B_0^*:

$$B_0^* = B_0^h - D_0^f. \tag{19}$$

- The economy NFA position is just the households' assets subtracted by firms' initial debt.

Equilibrium

- Combining the household's intertemporal budget constraint with firm's profits in periods 1 and 2, we arrive at the economy's resource constraint:

$$\underbrace{C_1 + \frac{C_2}{1+r_1} + I_1}_{\text{Domestic absorption}} = \underbrace{A_1 F(I_0) + \frac{A_2 F(I_1)}{1+r_1}}_{\text{Output}} + \underbrace{(1+r_0)B_0^*}_{\text{Initial wealth}}. \tag{20}$$

- We can recover the trade balance and current account as usual:

$$TB_t = Q_t - C_t - I_t \tag{21}$$

$$CA_t = TB_t + r_{t-1}B_{t-1}^* \tag{22}$$

Equilibrium

- An equilibrium in this production economy can then be defined as the solution of four endogenous objects: the allocations: $\{C_1, C_2, I_1\}$ and the price $\{r_1\}$ that satisfies:

$$r_1 = r^* \tag{23}$$

$$A_2 F'(I_1) = 1 + r_1 \tag{24}$$

$$\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = 1 + r_1 \tag{25}$$

$$C_1 + \frac{C_2}{1 + r_1} + I_1 = A_1 F(I_0) + \frac{A_2 F(I_1)}{1 + r_1} + (1 + r_0) B_0^* \tag{26}$$

- where the equations, in order, are: Small Open Economy assumption, investment demand, the Euler Equation, and the economy's intertemporal resource constraint.

- Given the functions U and F, we can solve the endogenous variables as functions of the exogenous variables.

Outline

The Current Account and Savings

- We have solved for equilibrium in the economy, which gives delivers consumption, investment and the interest rate.
- We can use this information to think about savings in the economy, and more specifically how the current account will fluctuate given different shocks.
- In particular, recall that there is no investment in period 2 $(I_2 = 0)$, and we are able to solve for $\{C_1, C_2, I_1, r_1\}$.
- Therefore, we can easily study changes in the trade balance, current account, and the net international investment position given our national account definitions from before.
- In doing so, we will also map out a savings schedule, which we can combine with the investment schedule to think about how the current account adjusts given different shocks to the economy.
- Recall: $CA = S - I$!

The Current Account and Savings

- We will focus on the savings decision in period 1. To link savings (and investment) in period 1 to the external account, recall some key equations:

$$Q_1 = A_1 F(I_0) \tag{27}$$
$$TB_1 = Q_1 - C_1 - I_1 \tag{28}$$
$$CA_1 = TB_1 + r_0 B_0^* \tag{29}$$
$$B_1^* = CA_1 + B_0^* \tag{30}$$

- Crucially, in looking at the above equations, only two variables are endogenous: C_1 and I_1. All other variables are pre-determined (time 0) or exogenous to the model (technology).
- Further, these variables are linked to the intertemporal price – i.e., the interest rate r_1.

The Savings Schedule

- Recall that savings is simply national income less consumption (in this case only private, since we have no government):

$$S_1 = Y_1 - C_1 \tag{31}$$
$$S_1 = \underbrace{Q_1(A_1) + r_0 B_0^*}_{Y_1} - C_1(r_1, A_1, A_2). \tag{32}$$

- Production is a function of a predetermined investment I_0 and the technological shock A_1: $Q_1(A_1) = A_1 F(I_0)$.
- Consumption on the other hand depends on r_1, A_1 A_2:

$$C_1 = C(\underset{-}{r_1}, \underset{+}{A_1}, \underset{+}{A_2}) \tag{33}$$

The Savings Schedule

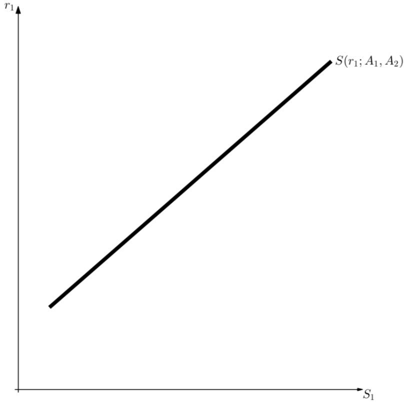
- Combining all the expressions, we have the savings is:

$$S_1 = Y_1(\underset{+}{A_1}) - C(\underset{-}{r_1}, \underset{+}{A_1}, \underset{+}{A_2}). \tag{34}$$

- It is easy to see that national savings is a increasing in r_1 and decreasing in A_2.
- But what about A_1? Recall that an increase in A_1 increases C_1 by less than output. Thus, national saving increases unambiguously with an increase in A_1.
- National saving can then be written as:

$$S_1 = S(\underset{+}{r_1}, \underset{+}{A_1}, \underset{-}{A_2}). \tag{35}$$

The Savings Schedule



Changes in technology A_1 and A_2 move the curve to the right and to the left.

The Current Account Schedule

- Recall the definition of CA:

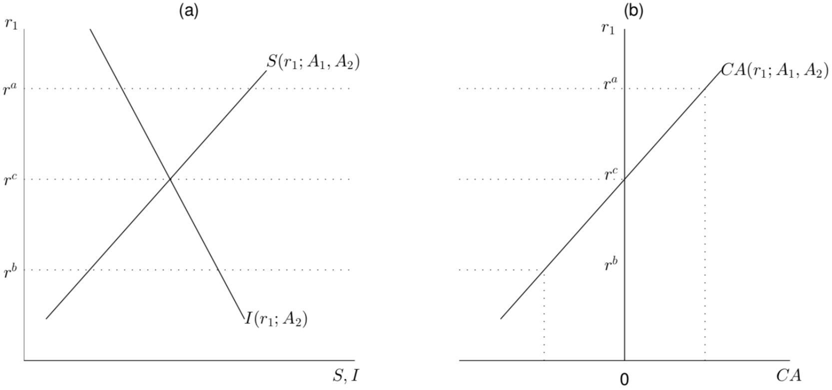
$$CA_1 = S_1 - I_1 \tag{36}$$

- Use the saving schedule and the investment schedule:

$$\begin{aligned} CA_1 &= S(\underset{+}{r_1}; \underset{+}{A_1}, \underset{-}{A_2}) - I(\underset{-}{r_1}; \underset{+}{A_2}) \\ &= CA(\underset{+}{r_1}; \underset{+}{A_1}, \underset{-}{A_2}) \end{aligned} \tag{37}$$

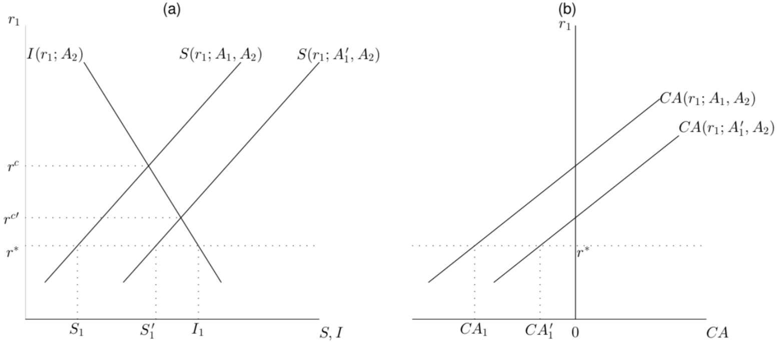
- This equation represents the current account schedule, which expresses the current account as an increasing function of the interest rate with productivity as shifters of this function.
- Note that $r_1 = r^*$, so we can interpret as the effect of the international interest rate on the current account.

The Current Account Schedule



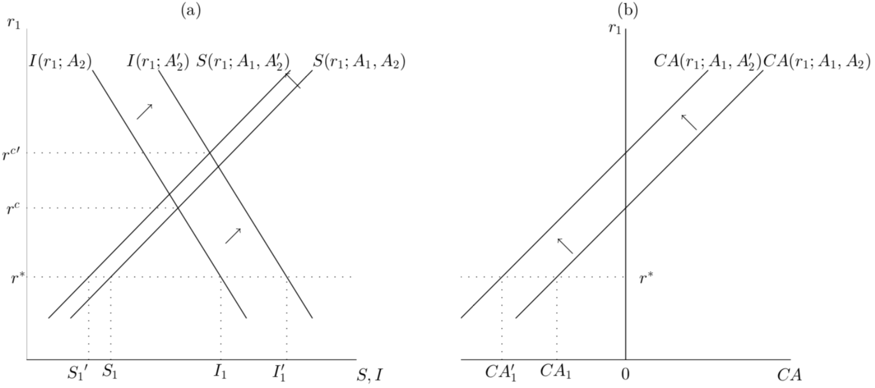
If r^* increases: savings will increase, investment will decrease and the CA rise.

Effect of a temporary increase in productivity



If A_1 increases: savings will increase, investment do not change and the CA rise.

Effect of an expected future increase in productivity



If A_2 increases: savings will decrease, investment will increase and the CA deteriorates.

Outline

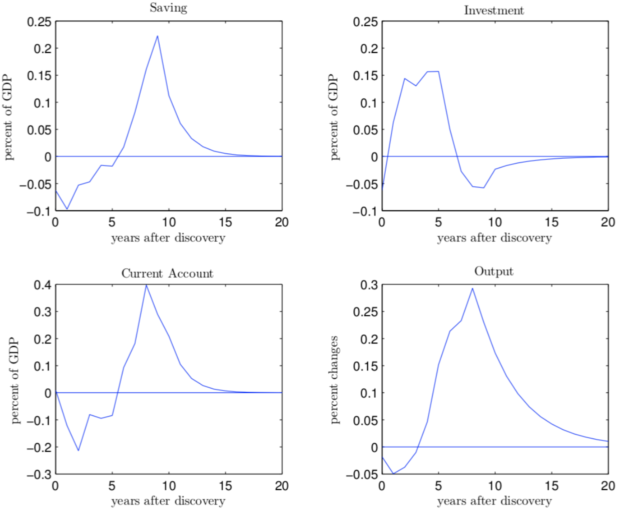
Application: Giant Oil Discoveries

- Are the predictions of the intertemporal production model of saving, investment, output, and current account determination studied in this chapter empirically compelling?
- To answer this question, we examine the dynamics triggered by giant oil discoveries.
- In the context of our model, news of a giant oil discovery can be interpreted as an anticipated increase in the productivity of capital, that is, as an anticipated increase in A_2
- The reason why a discovery is an anticipated productivity shock is that it takes time and extensive investment in oil production facilities to extract the oil and bring it to market.
- The average delay from discovery to production is estimated to be between 4 and 6 years.

Application: Giant Oil Discoveries

- Arezki, Ramey, and Sheng analyze the effects of giant oil discoveries using data from 180 countries over the period 1970 to 2012.
- They define a giant oil discovery as a discovery of an oil or gas field that contains at least 500 million barrels of ultimately recoverable oil equivalent.
- Giant oil discoveries are really big, with a median value of 9 percent of a year's GDP.
- The sample contains in total 371 giant oil discoveries, which took place in 64 different countries.
- The dynamic responses are estimated using dynamic panel model estimation with distributed lags.
- Essentially, one runs a regression of the variable of interest, say the current account, onto its own lags, and current and lagged values of the giant oil discovery, and other control variables, such as a constant and time and country fixed effects.

Empirical Evidence from Giant Oil Discoveries



Empirical Evidence from Giant Oil Discoveries

- Upon news of the giant oil discovery, investment experiences a boom that lasts for about 5 years.
- Saving declines and stays below normal for about 5 years, before rising sharply for several years.
- The current account deteriorates for 5 years and then experiences a reversal with a peak in year 8.
- Output is relatively stable until the fifth year, and then experiences a boom.
- The investment boom and the fall in saving and the current account last for roughly the delay from discovery to production typical in the oil industry.

Empirical Evidence from Giant Oil Discoveries

- The estimated empirical responses are consistent with the predictions of the theoretical model.
- The giant oil discovery induces oil companies to invest in the construction of drilling platforms.
- In addition, households anticipate higher profit income from the future exports of oil and as a result increase consumption and cut saving.
- Both the expansion in investment and the contraction in saving contribute to current account deficits in the initial years following the discovery.

Extension: Firm's Collateral Constraints

- We have thus far considered that financial markets are perfect. This assumption is far from reality.
- Both households and firms face constraints in their ability to borrow, regardless if they are willing to pay the prevailing interest rate.
- There is a large literature that examines why such constraints exist. Very often the underlying microfoundations are based on imperfect information.
- We will take these as given, and consider one particular case, where the firm faces collateral constraints, which limit their ability to borrow.

Extension: Firm's Collateral Constraints

What is collateral?

- Collateral is a property or other asset that a borrower offers as a way for a lender to secure the loan.
- If the borrower stops making the promised loan payments, the lender can seize the collateral to recoup its losses.
- For example, a house is the collateral for mortgage, or a factory and machinery is the collateral for a business loan.

Model with Collateral Constraint

- We will introduce a simple form of firm-level collateral constraints in period 1. Specifically, a firm can only borrow at most κ_1 units:

$$D_1^f \leq \kappa_1 \tag{38}$$

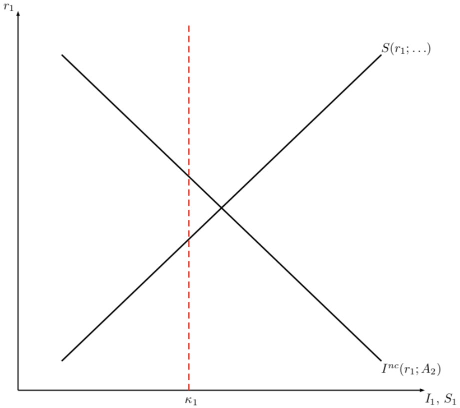
- As written, the constraint may or may not bind. If the constraint is not binding, investment decisions are identical to the model we have studies thus far.
- Define the non-binding level of investment as I_1^{nc}, then:

$$A_2 F'(I_1^{nc}) = 1 + r_1 \tag{39}$$

- If the constraint does bind, however, firms can only invest up to the maximum level of debt given the collateral constraint (i.e., κ_1). Therefore, the investment function is now

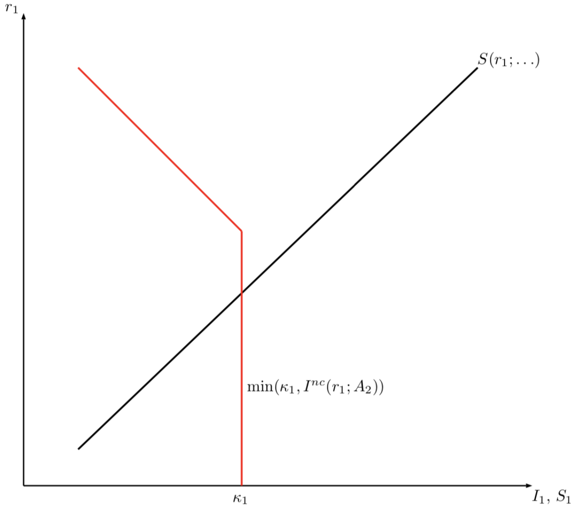
$$I_1 = \min\{\kappa_1, I_1^{nc}\} \tag{40}$$

Collateral Constraints, Investment, and Savings



We can interpret a decrease in κ_1 as a "financial shock" that tightens the collateral constraint.

Collateral Constraints, Investment, and Savings



Depending on the world interest rate, r^*, investment will either hit the collateral constraint or not.

Taking Stock

- We extended the 2-period model with production and investment. Firms invest up to a point at which the marginal product of capital equals the gross interest rate.
- Using the solution of the model, we shown that investment decreases with interest rate and savings increase with the interest rate.
- An increase in the world interest rate causes an improvement in the CA_1, an increase in saving, and a decrease in investment.
- The CA_1 improves with A_1, but deteriorates with A_2.
- We discussed an application on giant oil discoveries and an extension with collateral constraints.