

# Advanced Macroeconomics

Life Cycle Economies: Storesletten, Telmer & Yaron (2004, JME)

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# Introduction

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- **Goal:** introduce the incomplete markets framework in a OLG economy.
- Study a classic paper as an example: **Storesletten, Telmer & Yaron (2004): Consumption and risk sharing over the life cycle.**
- The life cycle structure is useful to study many questions where age interacts with inequality:
  - ▶ **Early age:** Education;
  - ▶ **Middle age:** Labor market;
  - ▶ **Old age:** Social security, health;

# Storesletten, Telmer & Yaron (2004): Motivation

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- Stylized Facts:

1. Inequality in consumption and earnings increase substantially during the life cycle;
2. The increase in inequality of consumption is less than earnings;
3. The increase is approximately linear.

- Can noninsurable idiosyncratic shocks to labor earnings explain this facts?
- What is the role of initial heterogeneity in comparison to earnings shocks during the life cycle?

# Storesletten, Telmer & Yaron (2004): Method

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- Estimate a rich labor earnings process using the PSID:
  - ▶ Individual fixed effects;
  - ▶ Persistent shocks;
  - ▶ Transitory shocks.
- Input the earnings process in an OLG model without consumption risks sharing.
  - ▶ General equilibrium pins down the level of wealth.
- Only two sources of insurance:
  - ▶ Self-insurance;
  - ▶ Pension system financed by labor tax.

# Empirical Evidence

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- **Data**

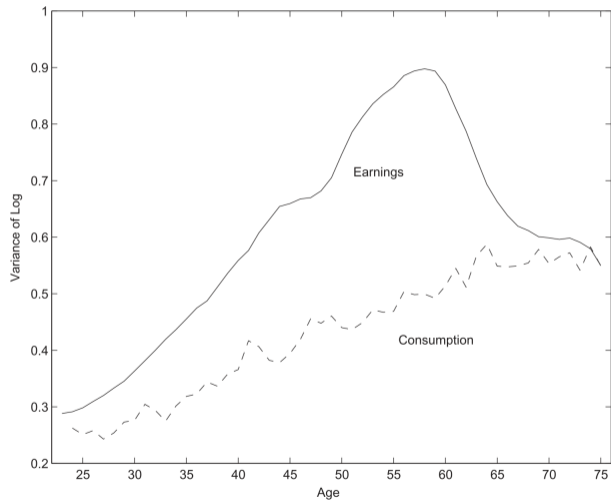
- ▶ **Panel Study of Income Dynamics (PSID, 1969-1992)**: Household survey; Panel data.  
Earnings: wage income before taxes, plus transfers.
- ▶ **Consumption Expenditure Survey (CEX, 1980-1990)**: Consumption survey;  
Consumption: nonmedical and nondurable expenditures on goods and services by urban U.S. households.

- Unit of study: household.

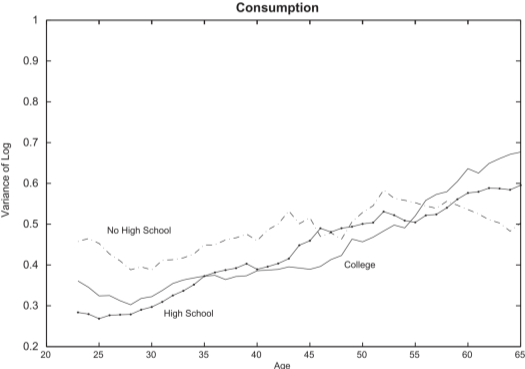
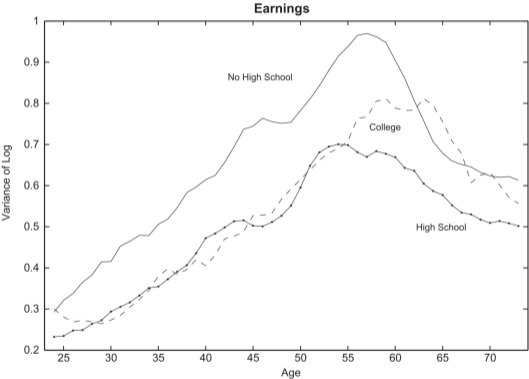
- Clean for cohort effects using a linear regression.

# Empirical Evidence

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# Empirical Evidence: by Education



# Earnings Stochastic Process

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- Let household  $i$  of age  $h$ . Denote the **residual** log of annual earnings as  $u_{ih}$  (i.e., log income with mean zero and net of cohort effects).
- The stochastic process of  $u_{ih}$  is defined as:

$$u_{ih} = \alpha_i + \epsilon_{ih} + z_{ih}$$

$$z_{ih} = \rho z_{i,h-1} + \eta_{ih}$$

where  $\alpha_i \sim N(0, \sigma_\alpha^2)$ ,  $\epsilon_{ih} \sim N(0, \sigma_\epsilon^2)$ ,  $\eta_{ih} \sim N(0, \sigma_\eta^2)$ , and  $z_{i0} = 0$ .

- Interpretation of each idiosyncratic shock:
  - ▶ **Fixed effect**,  $\alpha_i$ : Innate ability, early education investments, etc.
  - ▶ **Transitory shock**,  $\epsilon_{ih}$ : Earnings bonus, transitory health problems, etc.
  - ▶ **Persistent shock**,  $\eta_{ih}$ : Unemployment shocks with scarring effects, promotions, etc.



# Earnings Stochastic Process

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- At  $h = 0$  (age 22), the variance of  $u_{i0}$  is  $V(u_{i0}) = \sigma_\alpha^2 + \sigma_\epsilon^2 + \sigma_\eta^2$ .
- At  $h = 1$  (age 23), the variance of  $u_{i1}$  is  $V(u_{i1}) = \sigma_\alpha^2 + \sigma_\epsilon^2 + \sigma_\eta^2 + \rho^2 \sigma_\eta^2$ .
- The variance of  $u_{ih}$  for age  $h$ :

$$V(u_{ih}) = \sigma_\alpha^2 + \sigma_\epsilon^2 + \sigma_\eta^2 \sum_{j=0}^{h-1} \rho^{2j}.$$

- The variance of earnings increases during the life-cycle as persistent shocks accumulate!
- The rate of the increase depends on how persistent are the shocks:  $\rho$ .
  - ▶ If  $\rho = 1$ , shocks are permanent and their effects never fade out.

# Earnings Stochastic Process

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- The goal is to estimate the parameters:  $\rho$ ,  $\sigma_\eta^2$ ,  $\sigma_\alpha^2$ , and  $\sigma_\epsilon^2$ .
- Use GMM to estimate the parameters. Identification intuition (case  $\rho = 1$ ). Take the difference

$$\Delta u_{ih} = \Delta \epsilon_{ih-1} + \eta_{ih}$$

and use the moments:

$$V(\Delta u_{ih}) = 2\sigma_\epsilon^2 + \sigma_\eta^2$$

$$COV(\Delta u_{ih}, \Delta u_{ih+1}) = -\sigma_\epsilon^2$$

- ▶ To recover  $\sigma_\alpha^2$ , use  $\sigma_\eta^2$ ,  $\sigma_\epsilon^2$  and the variance of levels,  $V(u_{ih})$ .
- ▶ To estimate  $\rho$ , an extra time period is required so we need a panel of at least 4 time periods.
- ▶ STY actually use all moments in levels. The broad idea is similar.

# Model

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- The economy is populated by  $H$  overlapping generations. Denote  $\phi_h$  as the unconditional probability of surviving up to age  $h$ , preferences are:

$$\mathbb{E} \sum_{t=1}^H \beta^h \phi_h \frac{c_h^{1-\gamma}}{1-\gamma}, \quad \text{where } \beta \in (0, 1).$$

- Agents begin to work at 22 and, conditional on surviving, retire at 65. At 100 die with certainty.
- Technology:  $Y = ZK^\theta N^{1-\theta}$ .
  - ▶ Firms hire labor and rent capital at prices  $W$  and  $R$ .
  - ▶ Law of motion:  $K' = Y - C + (1 - \delta)K$ .
  - ▶ The economy has SS growth rates of  $g$ , so some variables must be normalized.

# Budget Constraint

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- Budget constraint of a working agent:

$$c_h + (1 + g)a'_h \leq a_h R / \xi_h + n_h (1 - \tau)W$$

where  $\tau$  is a labor tax, and  $\xi_h = \phi_h / \phi_{h-1}$  is the survivor's premium.

- The labor endowment process is given by:

$$\log n_h = \kappa_h + u_h$$

where  $\kappa_h$  are the age-profile earnings common to all agents, while  $u_h$  is the individual-specific stochastic process as defined before.

# Budget Constraint

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- Budget constraint of a retired agent:

$$c_h + (1 + g)a'_h \leq a_h R / \xi_h + B(\bar{n}_h)W$$

where  $B(\bar{n}_h)$  is the pension replacement rate that is a function of the average labor endowments over the life cycle,  $\bar{n}_h$ .

- The avg. labor endowment,  $\bar{n}_h$ , summarizes the social security contribution and evolves as following:

$$\bar{n}_{h+1} = \begin{cases} \bar{n}_h + n_h / I & \text{if working,} \\ \bar{n}_h & \text{if retired,} \end{cases}$$

where  $I$  is the number of years before retirement.

# Value Function

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- Let  $V_h$  denote the value function of an  $h$  years old agent. The value function of the agent is:

$$V_h(\alpha, z_h, \epsilon_h, a_h, \bar{n}_h) = \max_{a'_{h+1} \geq \underline{a}(\alpha, z, h)} \left\{ \frac{c_h^{1-\gamma}}{1-\gamma} + \hat{\beta} \xi_{h+1} \mathbb{E}_h[V(\alpha, z'_{h+1}, \epsilon'_{h+1}, a'_{h+1}, \bar{n}'_{h+1})] \right\}$$

s.t.

$$c_h + (1+g)a'_h = \begin{cases} a_h R / \xi_h + n_h(1-\tau)W & \text{if working,} \\ a_h R / \xi_h + B(\bar{n}_h)W & \text{if retired,} \end{cases}$$

where  $\hat{\beta} = \beta(1+g)^{1-\gamma}$  and  $\underline{a}(\alpha, z, h)$  is an age-dependent borrowing constraint.

- You can solve the value function using backward induction, as  $V_{H+1} = 0$  and  $a'_{H+1} = 0$ .

# Equilibrium

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- Let  $S = \{\alpha, z, \epsilon, a, \bar{n}, h\}$  be the state space.
- A stationary equilibrium is defined as prices,  $R$  and  $W$ ; a set of functions,  $\{V_h, a'_{h+1}\}_{h=1}^H$ ; aggregate capital stock  $K$  and labor supply  $N$ ; and a cross-sectional distribution  $\mu$  of agents across  $S$ , such that:
  - (a) Prices are given by the firm's marginal productivity of labor and capital;
  - (b) Functions  $\{V_h, a'_{h+1}\}_{h=1}^H$  solve the individual's problem;
  - (c) Given individual decisions, the distribution  $\mu$  is stationary;
  - (d) Pension tax satisfies the social security budget constraint:  $\int_S B(\bar{n})d\mu = N(1 - \tau)$ .
  - (e) Capital and labor market clears:  $K = \int_S a_h d\mu$  and  $N = \int_S n_h d\mu$ .

# Calibration

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- **Standard parameters:**  $\theta = 0.4$ ,  $\gamma = 2$ ,  $\delta = 0.109$ .
- **Stochastic process parameters:**  $(\rho, \sigma_{\eta}^2, \sigma_{\epsilon}^2, \sigma_{\alpha}^2, \kappa_h)$  estimated using PSID.
- $B(\bar{n}_h)$  replicates the pension system in the US.
- $\beta = 0.961$  matches wealth-to-income ratio of 3.1 in the US.



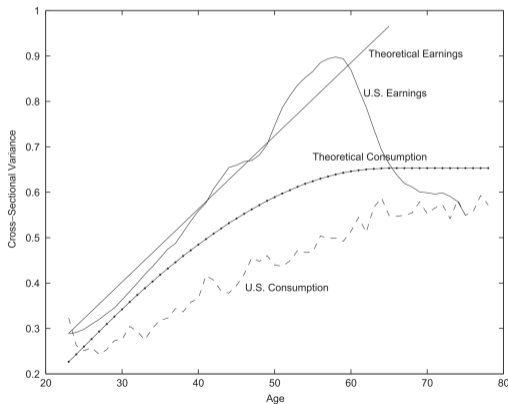
# Results

## Qualitatively Successful

- Consumption inequality is lower than earnings inequality;
- Earnings inequality increase faster than consumption inequality.

**Quantitative:** Consumption still a bit off.

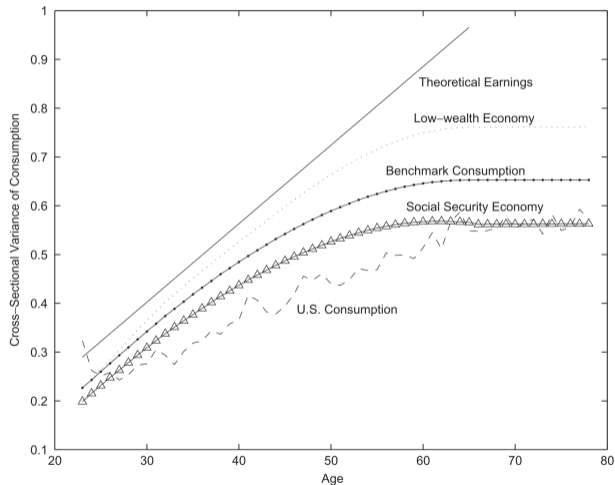
Figure: Model without Social Security ( $B(\bar{n}_h) = 0$ )



# Results

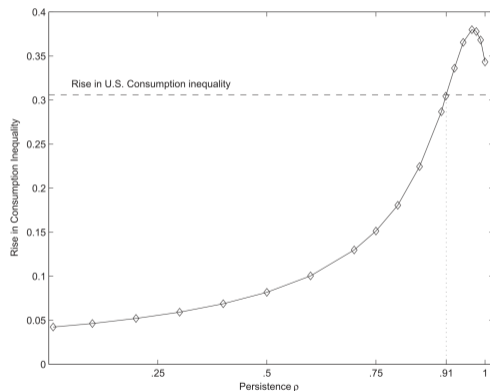
**Social Security:** it decreases consumption inequality, matches the data better;

**Importance of Wealth:** ↓ wealth-to-income ratio, ↓ the self-insurance and ↑ consumption inequality.



# What matters for Consumption Inequality?

- To generate enough consumption inequality, we need shocks to have enough persistence.
- Borrowing constraints and initial wealth inequality: matters for inequality between 23-29, but it is not very important later.



# Life-cycle shocks versus fixed effects

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## What type of inequality costs more for the agent?

- **Utilitarian measure:** how much consumption the agent is willing to forgo to live in a world without shocks?
- Let  $\psi$  the percentage consumption loss. Rewriting the utility function:

$$\mathbb{E} \sum_{t=1}^H \beta^h \phi_h \frac{[c_h(1-\psi)]^{1-\gamma}}{1-\gamma} = (1-\psi)^{1-\gamma} \mathbb{E} \sum_{t=1}^H \beta^h \phi_h \frac{c_h^{1-\gamma}}{1-\gamma} = (1-\psi)^{1-\gamma} \mathbb{E} V_1(\alpha, z, \epsilon, 0),$$

where  $\mathbb{E} V_1(\alpha, z, \epsilon, 0)$  is the average lifetime utility of a unborn agent (under the veil of ignorance).

- We can do the same thing for a model without risk, social security, etc.

## Life-cycle shocks versus fixed effects

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- Solve the model without risk and compute the expected VF at age 1:  $\mathbb{E}\hat{V}_1(\alpha, 0|\text{no risk})$ .
- What is  $\psi$  that equalizes expected utility in both worlds?

$$(1 - \psi)^{1-\gamma} \mathbb{E}\hat{V}_1(\alpha, 0|\text{no risk}) = \mathbb{E}V_1(\alpha, z, \epsilon, 0) \iff \psi = 1 - \left( \frac{\mathbb{E}V_1(\alpha, z, \epsilon, 0)}{\mathbb{E}\hat{V}_1(\alpha, 0|\text{no risk})} \right)^{1/(1-\gamma)}$$

- The **consumption equivalent variation** of each type of shock:
  - ▶  $\psi_{z,\epsilon} = 27.4\%$ .
  - ▶  $\psi_{\alpha} = 20.2\%$ .
- Shocks are costlier than ex-ante heterogeneity!

# Conclusion

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- Inequality in earnings and consumption increase during the life cycle.
- Persistent shocks are key to account for this regularity.
- Social security reduces welfare inequality.
- What other policies can achieve less welfare inequality?

## Where to go now?

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- **Pension System:** Conesa and Krueger (1999), Fuster et al (2007), McKiernan (2021).
- **Inequality over the Life cycle:** Huggett, Ventura and Yaron (2011), Guvenen, Kuruscu, Ozkan (2014).
- **Human Capital and Intergenerational Mobility:** Lochner and Monge-Naranjo (2011), Daruich (2020), Abbot et al (2019), Restuccia and Urrutia (2004).
- **Earnings Process:** De Nardi et al (2020), Guvenen et al (2021).
- **Welfare Policy:** Guner, Kaygusuz and Ventura (2021, WP) Low, Meghir and Pistaferri (2010, AER), Wellschmied (2021, QE).
- **Consumption Insurance:** Kaplan and Violante (2010), Blundell, Pistaferri and Preston (2008).
- **Marriage and Female Labor Supply:** Voena (2015), Attanasio, Low and Sanchez-Marcos (2008).
- **Old Age and Health Shocks:** many papers by Mariacristina Denardi and Eric French.