Advanced Macroeconomics
Problem Set 2

In this problem set, you should open and modify the python noteboook `pset2_blank.ipynb`. In both questions, you should consider the following heterogeneous agents model:

$$\begin{aligned} \max_{c_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\sigma}}{1-\sigma} \\ c_{it} + a_{it} = (1+r_t)a_{it-1} + Z_t s_{it} \\ a \geq \underline{a} \end{aligned}$$

where Z_t is the aggregate post-tax income. The market clearing conditions depend on the question.

1. **(Debt-Financed Fiscal Policy)** Consider the simple HANK model above. Interest rates are constant and Y_t demand-driven (i.e., prices are fully fixed so Y_t has to adjust to satisfy the market clearing equations). The goods and assets market clearing conditions are:

$$Y_t = C_t + G_t \quad \text{and} \quad A_t \equiv \int a_{it} di = B_t,$$

where $C_t \equiv \int c_{it} di$ and B_t is government debt. The government per-period budget constraint is:

$$T_t = (1+r_t)B_{t-1} + G_t - B_t.$$

Finally, we set after-tax income $Z_t = Y_t - T_t$. We will also set a representative agent (RA) and a two-agent (TA) version of the same model. The TA model has one agent that behaves like a representative agent and an agent that is hand-to-mouth. Let λ be the share of hand-to-mouth agent.

   (a) Set up the three models. Calibrate β so that both models have the same interest rate (the other parameters are in the notebook). Set $\lambda = 0.25$. Does β change if we change the share of hand-to-mouth agents?

   (b) Compute the impulse response of output and consumption to a fiscal policy shock (i.e., G_t) that is tax-financed. The shock follows an AR(1) process. Assume the shock has an initial size of 0.01 and a persistence of 0.8. Plot the response of output in all three models. In which model is the output response stronger? How does consumption respond in all models?

   (c) Let's now construct a debt-financed fiscal policy shock. This means you have to shock the government debt B_t at the same time as the G_t shock. The debt shock also follows an AR(1) process. Plot the impulse response function of output, consumption, taxes, and primary deficit ($G_t - T_t$) in the HA model for a low-persistence ($\rho_B = 0.5$) and a

high-persistence ($\rho_B = 0.8$) debt shock.[1] In which case is the response of output and consumption higher?

(d) Plot the impulse response function of the high-persistence debt-financed fiscal policy shock for the RA and the TA models. Change the share of hand-to-mouth to $\lambda = 0.75$. Compare the three cases with the response of the HA model.

2. **(Direct and Indirect Effects of the Monetary Policy)**. Consider the very simple HANK model defined above. The goods and assets market clearing are:

$$C_t \equiv \int c_{it} di = Y_t \qquad \text{and} \qquad A_t \equiv \int a_{it} di = 0.$$

There is no government, hence $Z_t = Y_t$. This implies the following fixed point: $\mathcal{C}_t(\{r_s, Y_s\}) = Y_t$.

We will consider a "demand-driven" (as if prices were fully fixed). In this case, a shock in the sequence of interest rate $\{r_s\}$ will require a change in the aggregate output Y_t.

As in the IKC model, the linearized response of output implies:

$$d\mathbb{Y} = \underbrace{\mathbb{M}^r dr}_{\text{direct effects}} + \underbrace{\mathbb{M} d\mathbb{Y}}_{\text{indirect effects}},$$

where $\mathbb{M}^r \equiv \partial \mathcal{C}_t / \partial r_s$ is the jacobian capturing the direct effect of the interest rate on $\mathcal{C}$.

The goal is to use the SSJ to decompose the direct and indirect effects of the monetary policy as in Kaplan-Moll-Violante (2018). We will do it in the HA model, as well as in the RA model.

(a) Set up two models. The first represents the heterogeneous-agent model from above. The second is a representative-agent version of the same model. Calibrate β so both models have the same interest rate (the other parameters are in the notebook). Which model has the highest β?

(b) Compute the impulse response of the output to an interest shock (i.e., the 'total effect' $d\mathbb{Y}$). The shock follows an AR(1) process. Assume the shock has an initial size of -0.01 and a persistence of 0.7. Plot the response of output in both models. In which model is the output response stronger?

(c) Compute the Jacobians $\mathbb{M}^r$ and $\mathbb{M}$. Use these Jacobians to decompose the total effect of the interest rate into direct and indirect effects in both models. Plot the effects and interpret them.

[1] Recall to keep the same persistence of the fiscal policy shock.