Advanced Macroeconomics
Problem Set 1

1. **(The Basics of the Consumption-Savings Problem)**. Consider the following household problem in dynamic programming form:

$$
\begin{aligned}
V(a,s) = \max_{a' \geq -\phi} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}[V(a',s')|s] \right\} \\
c + a' = (1+r)a + w \exp\{s\} \\
s_t = \rho s_{t-1} + \sigma \varepsilon_t, \qquad \text{where } \varepsilon \sim N(0,1).
\end{aligned}
$$

   The problem yields the household asset and consumption policy functions: g_a and g_c.

   (a) Discretize the labor income stochastic process using the Rouwenhorst and the Tauchen method. Use different number of grid points ($n_s = 3; n_s = 11, n_s = 21$) different persistence ($\rho = 0; \rho = 0.8; \rho = 0.99$) and standard deviation of the innovation ($\sigma = 0.1; \sigma = 1.0$).

   Let $\mu_{s,t}$ the probability distribution of individuals across states g_S. Write a function that simulates the distributions $\mu_{s,t}$ using the Markov Chain of stochastic process for 1000 periods starting from a distribution where all the individuals are in the middle grid point. Compare the variance (i.e., the inequality) of the invariant distribution using all methods, number of grid points, persistence and st. deviation.

   (b) Discretize the asset grid between $a_1 = 0.0$ and $a_{max} = 250.0$, and use $n_A = 300$ grid points. Include more points closer to the lower bound (see the lecture notes). Discretize the income stochastic process using the Rouwenhorst method with $n_s = 7$, $\rho = 0.9$, and $\sigma = 0.1$. Suppose $\beta = 0.96$, $\gamma = 2.0$, $r = 0.03$ and $w = 1$.

   Now solve the consumption-savings problem using the endogenous grid method. Plot the consumption and the savings policy functions in the asset space for the lowest and the highest s. Increase the persistence and the standard deviation of the innovations: $\rho = 0.99$ and $\sigma = 1.0$ (one at a time). How does the savings decision changes when the household has few asset holdings? Does the space of $g_a = 0.0$ increases or decreases when we change ρ and σ?

2. **(Solving the Aiyagari Model)**. Consider the standard Aiyagari economy. The aggregate production function is given by the Cobb-Douglas: $Y_t = K_t^{\alpha} L_t^{1-\alpha}$. Capital depreciates at rate δ. The household problem is defined as in the previous question.

   The stationary competitive equilibrium is defined by the prices w and r; policy functions for the household $g_a(a,s) = a'$ and $g_c(a,s) = c$; firm's choice K and L; and, a stationary distribution λ such that the asset market is in equilibrium:

$$
\int_{A \times S} a d\lambda(a,s;r) = K(r);
$$

the policy functions are the solution of the household problem given prices; capital demand, K, solves the firm's problem given prices; and the distribution, λ , is stationary given the household policy function, g_a.

(a) Outline a computational algorithm to find a stationary competitive equilibrium of the model.

(b) Solve the Aiyagari model in the computer using the following parameters: $\alpha = 0.33$, $\delta = 0.05$, $\beta = 0.96$, $\gamma = 2.0$, $\phi = 0.0$. The labor shock is discretized using a three states Markov-chain[1]

$$g_S = \begin{bmatrix} -0.32 \\ 0.00 \\ 0.32 \end{bmatrix}, \qquad \text{and} \qquad \Pi = \begin{bmatrix} 0.9025 & 0.095 & 0.0025 \\ 0.0475 & 0.905 & 0.0475 \\ 0.0025 & 0.095 & 0.9025 \end{bmatrix}.$$

The asset grid is discretized between $a_1 = -\phi$ and $a_{max} = 250.0$ (the number of grid points depend on the method used). After you solve the model, show: (i) the equilibrium values of r, w and K; (ii) the fraction of constrained households (at grid point a_1 in the invariant distribution); (iii) the marginal wealth distributions of s_1 and s_3; (iv) a measure of inequality of the wealth and income distribution (variance, Gini, or your preferred measure).

(c) Solve the model again for a looser borrowing constraint: $\phi = 5.0$ and $\phi = 10.0$. Compute the same statistics again. How inequality has changed in the model economy?

(d) Suppose there is higher inequality in the labor market:

$$g_S = \begin{bmatrix} -0.4 \\ 0.00 \\ 0.4 \end{bmatrix}$$

but keep $\phi = 0.0$ and the same Π (labor market *risk* is still the same). Solve the model and re-compute the statistics.

3. **(Universal Basic Income in the Aiyagari Model)**. Consider the standard Aiyagari model presented in the previous question. The parameter's values are the same as the previous question.

We model Universal Basic Income (UBI) as a lump-sum transfer: households are subject to a tax τ_l on their labor income, and this labor tax is used to finance a lump-sum transfer, T, (the UBI) to all households. The household budget constraint reads:

$$c_t + a_{t+1} = (1 + r_t)a_t + w_t \exp\{s_t\}(1 - \tau_l) + T_t.$$

The government balances its budget every period. For simplicity, we assume that the government does not run a public deficit. The government budget constraint reads:

$$T_t = \tau_l w_t L_t.$$

[1] These values are similar to a discretization of the AR(1) with $\rho = 0.9$ and $\sigma = 0.1$ using the Rouwenhorst method.

To solve the model in the computer, you must do the following changes in the algorithm: (i) after you have computed the wage w using your initial guess, you should compute the transfer, T, using the government budget constraint; (ii) you should modify the household budget constraint accordingly to take into account transfer/tax. Note that T is endogenously defined in equilibrium.

(a) Solve the model with the following labor tax: $\tau_l = 0.0$, $\tau_l = 0.05$, $\tau_l = 0.1$, and $\tau_l = 0.2$. Compute measures of inequality for gross income (pre-taxes and transfers), net income (post-taxes and transfers), and wealth.

(b) How did the introduction of the tax and the transfer change the equilibrium wage and interest rate? Intuitively, argue why the equilibrium prices are changing even though labor supply is fully inelastic.