Advanced Macroeconomics

Baseline HANK Model and Fiscal Policy in HANK

Tomás R. Martinez

INSPER

References

- Auclert, Rognlie and Straub (2018, NBER WP)*. The Intertemporal Keynesian Cross.
- Also check their NBER summer course notes here.
- Complementary reference: Hagedorn, Manovskii, and Mitman (2019, WP).

Introduction

- Let's introduce a canonical HANK model.
- What is a canonical HANK model? Many models out there.
- New set of moments are key for the results $\Rightarrow$ **Intertemporal Marginal Propensities to Consume** (iMPCs).
  - What the data of iMPCs look like?
  - What kind of models match the data?
  - Heterogeneous Agents (HA), Two Agents (TA), Representative Agent (RA)?

Fiscal Policy

- What is the effect of an increase in **government spending**?
  - Does modeling HA-agents matter?
  - Should the fiscal policy be **deficit-financed** or should the government balance its budget all periods?
- What is the importance of government liquidity for the MPCs?
- Should we use progressive taxation or lump-sum taxes to finance?
- How fiscal policy interacts with monetary policy?

A General Model

- Unit mass of individuals that live for $t = 1, .., \infty$.
- There is NO aggregate uncertainty, but agents may be subject to idiosyncratic shocks.
  - Idiosyncratic ability state e follows a Markov process with transition matrix Π.
  - Stationary distribution of state e is $\pi(e)$, average ability is normalized to one, i.e., $\sum_e \pi(e)e = 1$.
- Asset markets may or may not be complete, and There could be many assets with different liquidity.
- Governments may carry debt but must satisfy its intertemporal budget constraint.
- Flexible prices, but wage rigidity.
- **Simplifications:** no investment/capital, passive monetary policy.

Household Problem

- Household i enjoys consumption and gets disutility from labor:

$$
\begin{aligned}
\max \quad & \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ u(c_{it}) - v(n_{it}) \right\} \\
\text{s.t.} \quad & c_{it} + \sum_j a_{it}^j = z_{it} + (1 + r_{t-1}) \sum_j a_{it-1}^j \\
& a_{it}^j \in \mathcal{A}_{it}^j
\end{aligned}
$$

where z_{it} is the after-tax income and can capture progressive taxation:

$$
z_{it} \equiv \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it} \right)^{1-\lambda}
$$

- Note that the structure allows different assets j and a general asset-market structure, $\mathcal{A}_{it}^j$ (incomplete markets, different liquidity, etc).

Wage Rigidity

- Prices are flexible, but wages are sticky (see Erceg et al (2000) or Galí's book Chapt. 6). Introduce rigidity in layers so all HH work same number of hours $n_{it} = N_t$.

- There is a continuum of symmetric unions $k \in [0, 1]$.
  - Every worker i sells n_{ikt} hours to union k.
  - Each union aggregates efficient units of work into a union-specific task: $N_{kt} = \int e_{it} n_{ikt} di$.

- A competitive labor packer then package these tasks into aggregate employment using the CES:

$$N_t = \left(\int_k N_{kt}^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$$

  - The packer sells N_t to the aggregate firm that produces the final good.

Wage Rigidity: Packers

- The labor packer's demand tasks from the unions. The problem:

$$\max_{N_{kt}} \quad W_t N_t - \int W_{kt} N_{kt} dk \qquad \text{s.t.} \quad N_t = \left(\int_k N_{kt}^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$$

- Solution implies the following demand for union tasks and wage index:

$$N_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} N_t, \qquad \text{and} \qquad W_t = \left(\int W_{kt}^{1-\epsilon} dk \right)^{1/(1-\epsilon)}.$$

Wage Rigidity: Unions

- Unions set wages W_{kt} taking as given demand for their tasks N_{kt}.
- Workers do not like wage adjustments, so unions decide the wages to maximize discounted average utility of the workers subject to adjustment costs:

$$\max_{\{W_{kt+\tau}\}} \sum_{\tau \geq 0} \beta^{t+\tau} \left(\int \{u(c_{it+\tau}) - v(n_{it+\tau})\} d\Psi_{it+\tau} - \frac{\psi}{2} \left(\frac{W_{kt+\tau}}{W_{kt+\tau-1}} - 1 \right)^2 \right)$$

subject to

$$N_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} N_t \qquad \text{and HH budget constraint.}$$

New Keynesian Phillips Curve

- After some boring derivations here, since unions are symmetric, we can show:
  - All unions set the same wage, $W_{kt} = W_t$;
  - All HH work the same number of hours;

- It implies a non-linear **New Keynesian (Wage) Phillips Curve**:

$$\pi_t^w(1+\pi_t^w) = \frac{\epsilon}{\psi} \int N_t \left\{ v'(n_{it}) - \frac{(\epsilon-1)}{\epsilon} \frac{\partial z_{it}}{\partial n_{it}} u'(c_{it}) \right\} d\Psi_{it} + \beta \pi_{t+1}^w (1+\pi_{t+1}^w)$$

  - Conditional on future wage inflation, unions set higher nominal wages when MRS between n_{it} and c_{it} exceeds a marked-down average of mg. after-tax income from extra hours.
  - In the absence of rigidity: $v'(n_{it}) = \dfrac{(\epsilon-1)}{\epsilon} \dfrac{\partial z_{it}}{\partial n_{it}} u'(c_{it})$

Production Function

- Let X_t be the TFP. Assume no capital and CRS, aggregate production is given by:

$$Y_t = X_t N_t$$

- Due to perfect competition and flexible prices, the final goods price is given by:

$$P_t = \frac{W_t}{X_t} \quad \Rightarrow \quad \frac{W_t}{P_t} = X_t.$$

- Assume $X_{ss} = 1$, so in absence of TFP shocks, real wage is equal to one.

- Goods inflation $\pi_t =$ wage inflation, π_t^w, minus TFP growth.

Government Fiscal Policy

- Let be B_t the amount of gov. bonds. The government budget constraint:

$$B_t = (1 + r_{t-1})B_{t-1} + G_t - T_t$$

- Iterating and imposing a no-Ponzi scheme, we get the gov. intertemporal BC:

$$(1 + r_{t-1})B_{t-1} = \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1 + r_s} \right) (T_t - G_t)$$

- Aggregate tax revenue adjusts through τ_t according to:

$$T_t = \int \left[\frac{W_t}{P_t} e_{it} n_{it} - \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it} \right)^{1-\lambda} \right] di$$

Monetary Policy

- Assume no monetary shocks and that monetary policy follows a **real rate rule**.

- Equivalent to Taylor rule with coefficient, $\phi_\pi = 1$, on inflation.

$$r_t = r_{ss} + \varepsilon_t \qquad \Longleftrightarrow \qquad i_t = r_{ss} + \pi_t + \varepsilon_t$$

- Since there are no monetary shocks, $\varepsilon_t = 0$, by the Fisher equation implies a constant interest rate equal to the flexible-price steady-state interest rate r_{ss}.

$$r_t = i_t - \pi_t \qquad \Longrightarrow \qquad r_t = r_{ss} \qquad \text{for all } t = 0, \ldots\infty$$

- Intuitively, the nominal interest rates rise exactly enough to offset the (expected) inflation.
    - It brings tractability and allows the analysis to focus on forces orthogonal to monetary policy.

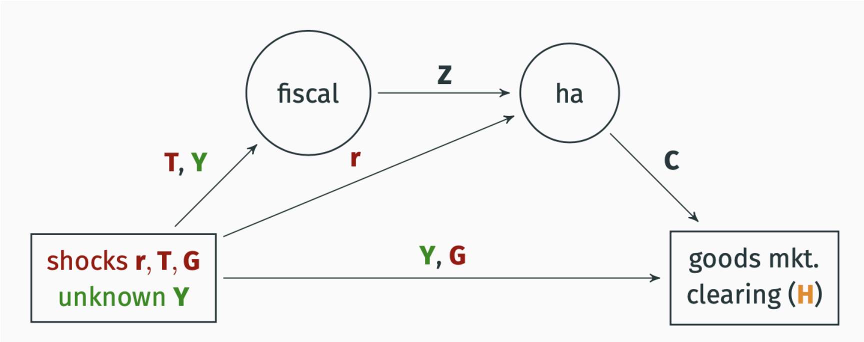
Equilibrium

- Given initial nominal wage W_{-1}, gov. debt B_{-1}, distribution $\Psi_{-1}(\{a^j, e\})$, and exogenous sequences for fiscal policy $\{G_t, T_t\}$, equilibrium is a path for prices, aggregates and individual allocations s.t agents maximize, policies are satisfied and **goods and bond market clear**:

$$G_t + \underbrace{\int c_t(\{a^j\}, e) d\Psi_t}_{C_t} = Y_t$$

$$\sum_j \int a^j d\Psi_t = B_t$$

Equilibrium: DAGs



- Goods mkt. clearing: $H \equiv C + G - Y$

Aggregate Consumption Function

- Let Z_t be the aggregate after-tax income:

$$Z_t \equiv \int z_{it} di = \tau_t N_t^{1-\lambda} \int e_{it}^{1-\lambda} di$$

- Individual after-tax income is a fraction of the aggregate:

$$z_{it} = \frac{e_{it}^{1-\lambda}}{\int e_{st}^{1-\lambda} ds} Z_t$$

- Given that r is constant and z_{it} is proportional to aggregate income Z_t, the individual policy rules $\{c_t, a_t^j\}$ is **entirely determined by the sequence of $\{Z_t\}$**.

Aggregate Consumption Function

- The **aggregate consumption function** is the aggregate of individual policies:

$$\int_i c_{it} di = C_t(\{Z_s\}) = C_t(\{Y_s - T_s\})$$

- Note that C_t depends on the **sequence** of $\{Z_s\}_{s=0}^{\infty} \Rightarrow C_t(Z_0, Z_1, ...)$.

- C_t encapsulates the complex interactions between heterogeneity, macroeconomic aggregates, and wealth distribution.
  - It is **forward-looking** (from the Euler Equation).
  - It also is **backward-looking** (from the distribution and HH budget constraint).

- The consumption function will be different for each model (HA, RA, TA).

The Keynesian Cross

- The consumption function implies a **Keynesian-Cross** type of equation:

$$Y_t = C_t(\{Y_s - T_s\}) + G_t.$$

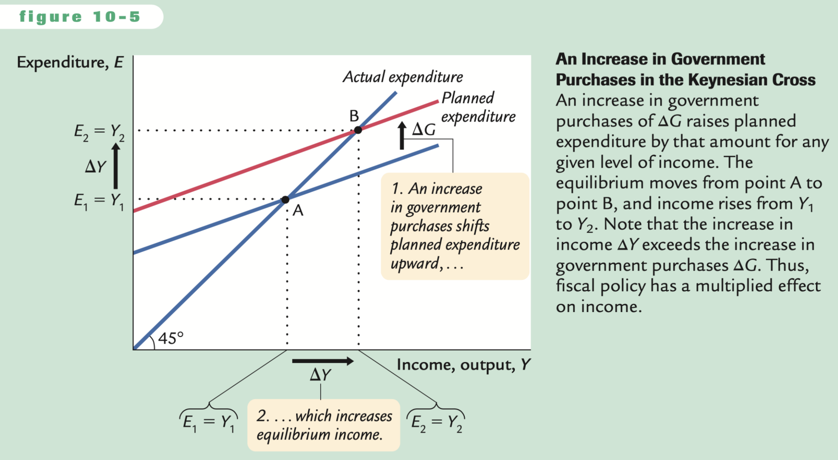
- Reminds you something? Recall your undergrad macro 1:

$$Y = C(Y - T) + G \qquad \text{where} \qquad C(Y - T) = c_0 + mpc \times (Y - T).$$

- The difference is that the power of fiscal policy depends not only on the current marginal propensity to consume but on the future and past mpc's as well.

  $\Longrightarrow$ Intertemporal mpc $(iMPC)$!

Undergraduate Keynesian Cross



- The **intertemporal Keynesian cross** is the same... just in vectors!

Intertemporal MPCs

- What is the effect of fiscal policy (i.e., G_t and T_t) on output? The goods mkt. clearing contains all the complexity of GE.

- Totally differentiating, we get the first-order response of output to changes in fiscal policy:

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t}{\partial Z_s}(dY_s - dT_s)$$

- The **intertemporal MPCs** represent how much consumption at t responds to a change in income at s:

$$M_{t,s} \equiv \frac{\partial C_t}{\partial Z_s}$$

- Since BC holds, all income is eventually spent, which implies: $\sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^{t-s}} = 1$. Proof

The Intertemporal Keynesian Cross

- Collect all the $M_{t,s}$ as the elements of a matrix $\mathbf{M}_{T\times S}$. Let the vectors represent the time sequences: $d\mathbf{Y} \equiv (dY_0, dY_1, ...)'$ (similarly for $d\mathbf{G}$ and $d\mathbf{T}$).
- If the response of output $d\mathbf{Y}$ to a fiscal policy shock $\{d\mathbf{G},\ d\mathbf{T}\}$ exists, it solves the **intertemporal Keynesian cross**:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- Let $\mathcal{M}$ some linear map that ensures $dY_t \to 0$ as $t \to \infty$, the solution is

$$d\mathbf{Y} = \mathcal{M}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$$

  There may be several $\mathcal{M}$ that solve for the linear map (indeterminacy). They restrict attention to $\lim_{\to\infty} dY_t \to 0$.

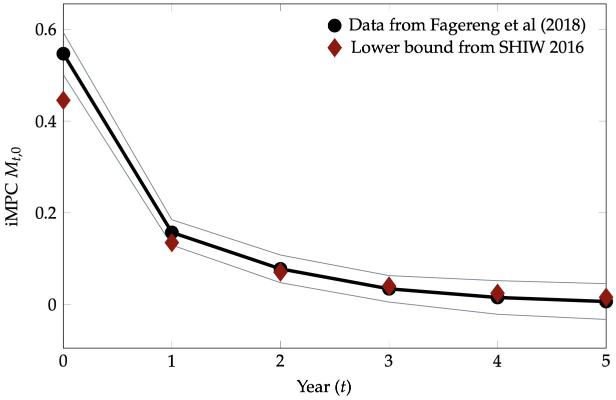
The Intertemporal Keynesian Cross

- The iMPC matrix is a **sufficient statistic**:
  - The entire complexity of the model is in $\mathbf{M}$.
  - The response of Y to fiscal policy shocks is in $\mathbf{M}$.
- There is a "correct" $\mathbf{M}$ out there in the data from the real world (it is just very hard to measure).
- It was possible to derive the "simple" intertemporal Keynesian cross given the many simplified assumptions.
  - Extensions: alternative tax incidence, durable goods, investment.
  - Limitations: passive monetary policy, sticky prices.

Which model matches the iMPC?

- Data on iMPC is hard to get. We usually only observe the first column $M_{t,0}$ for $t = 0, 1 \ldots$

Figure 1: iMPCs in the Norwegian and Italian data.



The iMPCs of the Representative Agent Model

- Suppose $\beta(1+r) = 1$, iterating the budget constraint and using the EE, the consumption function of the RA is:

$$C_t = (1-\beta)\sum_{s=0}^{\infty} \beta^s Z_s + r a_{-1}.$$

Proof

- Since $M_{t,s} = \frac{\partial C_t}{\partial Z_s} = (1-\beta)\beta^s$, the iMPC matrix is:

$$\mathbf{M}^{RA} = \begin{bmatrix} 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \dots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \dots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The iMPCs of the Two Agent Model

- A fraction μ are hand-to-mouth agents (HTM), $1-\mu$ are permanent income agents (PIH).
- Consumption function of each type of agent:

$$c_t^{PIH} = (1-\beta)\sum_{s=0}^{\infty}\beta^s Z_s + ra_{-1}, \qquad \text{and} \qquad c_t^{HTM} = Z_t$$

- Aggregate consumption function: $C_t = (1-\mu)c_t^{PIH} + \mu c_t^{HTM}$.
- The iMPC matrix is just a linear combination of both:

$$\mathbf{M}^{TA} = (1-\mu)\mathbf{M}^{RA} + \mu\mathbf{I}$$

- An useful extension is to introduce bonds/wealth in the utility function to mimic incomplete markets (TABU).

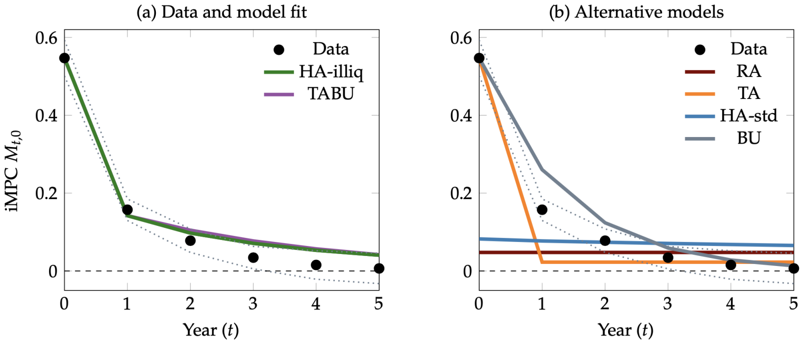
Which model matches the iMPC?

Table 1: Calibrating the benchmark models.

Parameters	Description	Values					
		HA-illiq	RA	TA	HA-std	BU	TABU
ν	Elasticity of intertemporal substitution	0.5	(same across all models)				
ϕ	Frisch elasticity of labor supply	1	(same across all models)				
r	Real interest rate	5%	(same across all models)				
λ	Retention function curvature	0.181	(same across all models)				
G/Y	Government spending to GDP	0.2	(same across all models)				
A/Z	Wealth to after-tax income ratio	8.2	(same across all models)				
β	Discount factor	0.80	0.95	0.95	0.92	0.90	0.90
B/Z	Liquid assets to after-tax income	0.26	8.2	8.2	8.2	8.2	8.2
$\underline{a}$	Borrowing constraint	0			0		
μ	Share of hand-to-mouth households			52%			36%

Which model matches the iMPC?

Figure 2: iMPCs in the Norwegian data and several models.



- HA with low liquidity (tight borrowing constraints or multiple illiquid assets) and TABU fit the data better.

Fiscal Policy

- Focus on two types of multipliers:
  - **Impact Multiplier:** dY_0/dG_0, and **Cumulative Multiplier:** $\frac{\sum_{t=0}^{\infty}(1+r)^{-1}dY_t}{\sum_{t=0}^{\infty}(1+r)^{-1}dG_t}$.
- Benchmark: Balanced budget multiplier $d\mathbf{G} = d\mathbf{T}$.
  - **Fiscal multiplier is always one:** $d\mathbf{Y} = d\mathbf{G}$.
  - Proof is trivial, $d\mathbf{Y} = d\mathbf{G}$ is the only solution of the iKC:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

  - **Intuition**: the increase in pretax income exactly offsets the increase in taxes for every household at every date and state.

Deficit Financed Fiscal Policy

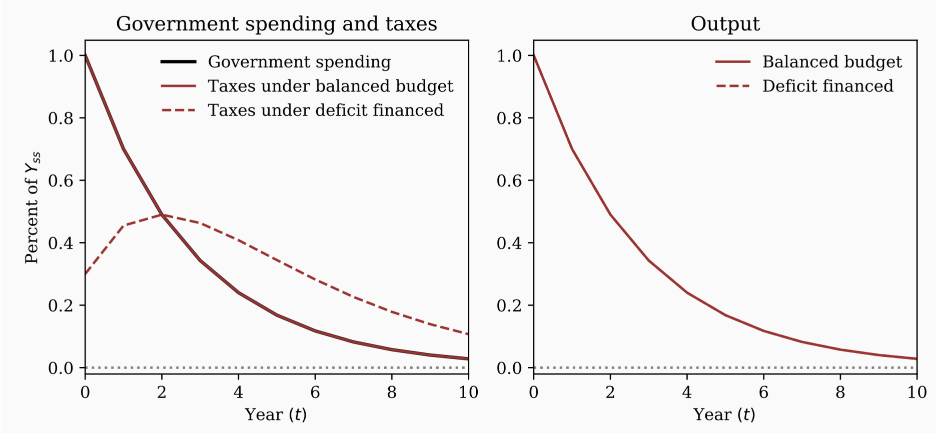
- Suppose a change in fiscal policy is financed with a deficit, i.e $d\mathbf{G} \neq d\mathbf{T}$. Then:

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T})}_{d\mathbf{C}}$$

- The change in consumption $d\mathbf{C}$ depends on the **path of primary deficits** $(d\mathbf{G} - d\mathbf{T})$.
- Crucial **interaction** between the iMPC matrix $\mathbf{M}$ and the primary deficit.
  - Different models have different $\mathbf{M}$.
  - May be worth running a deficit precisely at the time when iMPC is large.

Fiscal Policy in Representative Agent Model

- In the RA, $d\mathbf{Y} = d\mathbf{G}$ irrespective of $d\mathbf{T}$. Impact and cumulative multipliers are equal to 1.
  - **Intuition:** Since Ricardian Equivalence holds any policy is equivalent to a balanced budget.
  - This result may break with other types of monetary rules, ZLB, etc (Woodford, 2011).



Fiscal Policy in Two Agent Model

- In the TA model, the iKC equation is given by (see paper):

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu}(d\mathbf{G} - d\mathbf{T})$$

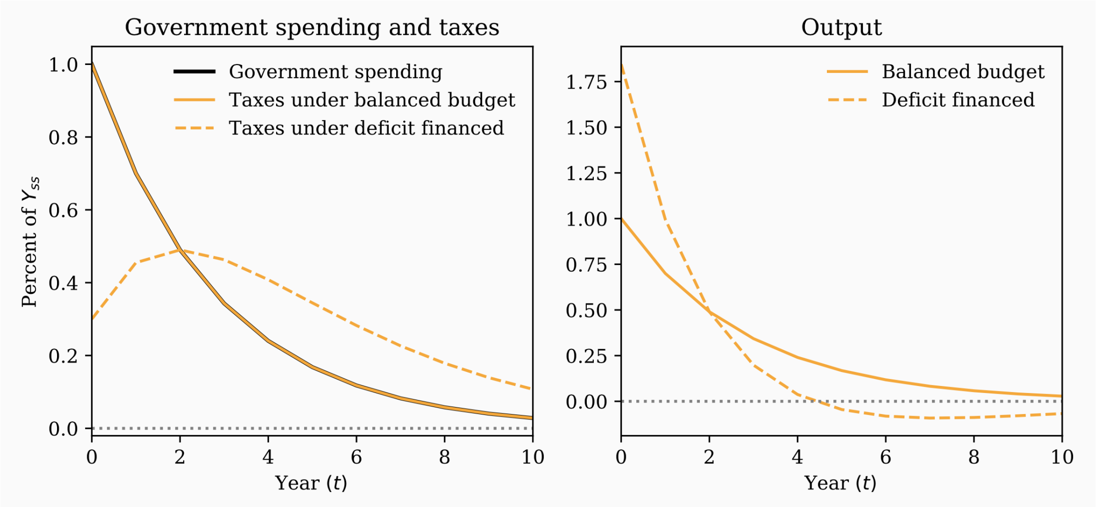
- Only **current deficit** matters.
  - The impact multiplier is a function of the share of HTM agents and the current deficit

$$\frac{1}{1-\mu} - \frac{\mu}{1-\mu}\frac{dT_0}{dG_0}$$

  - Cumulative multiplier is equal to one since consumption declines as soon as deficits are turned into surpluses.

- Model behaves remarkably similarly to static (undergrad) Keynesian cross.

Fiscal Policy in Two Agent Model

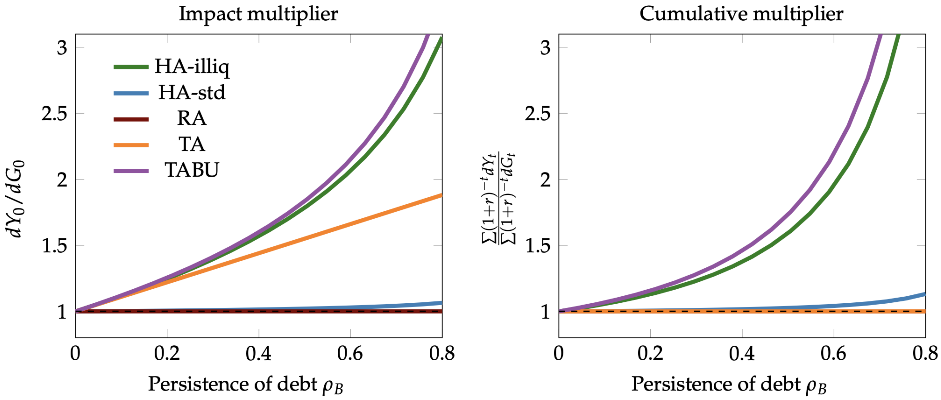


Fiscal Policy in the Benchmark Cases

- Suppose that government spending declines at a rate, $dG_t = \rho_G^t$.
- Taxes are chosen such that the path of public debt is given by: $dB_t = \rho_B(dB_{t-1} - dG_t)$.
  - Greater $\rho_B > 0$ leads to greater deficit.
  - If $\rho_B = 0$ policy keeps a balanced budget.
- Fiscal policy in HA agents can generate (deficit-financed) cumulative multipliers well above 1.
  - Intuition from zero-liquidity HA model (see notes).
  - Multiplier is a combination of the TA model, but with additional anticipatory and backward-looking terms.

Fiscal Policy in the Benchmark Cases

Figure 4: Multipliers across the benchmark models.

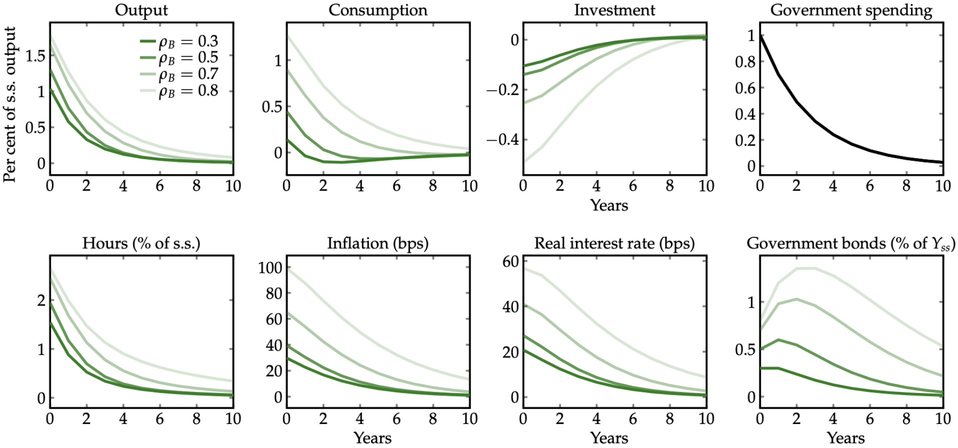


- The higher ρ_B, the higher is the multiplier.

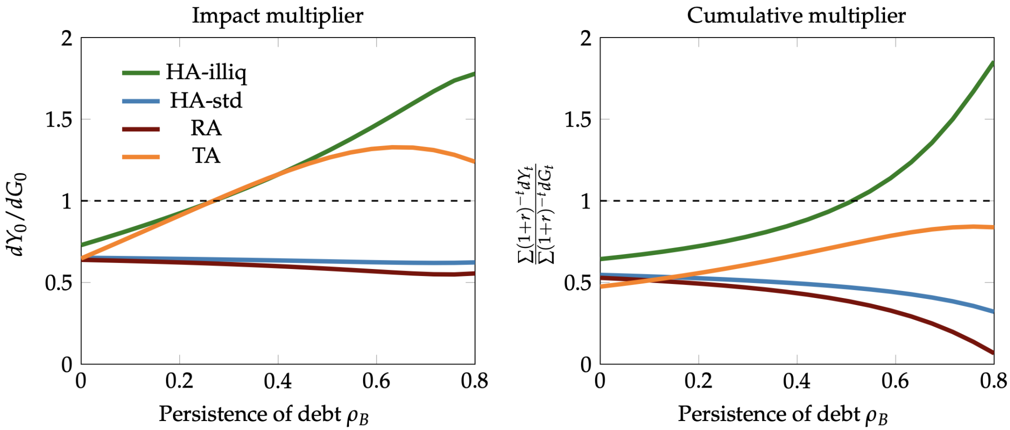
Fiscal Policy in the Quantitative Model

- Benchmark models kept the "supply side" simple to focus on iMPC.
- Compare with the full quantitative model:
    - Capital adjustment shocks;
    - Sticky prices;
    - Portfolio decision;
    - Monetary policy following a Taylor rule.
- The magnitude is smaller, but similar results hold (deficit-financed fiscal policy is stronger).
    - The supply side crowds out part of the effect $\Rightarrow \uparrow r$ and $\downarrow I$.

Fiscal Policy in the Quantitative Model



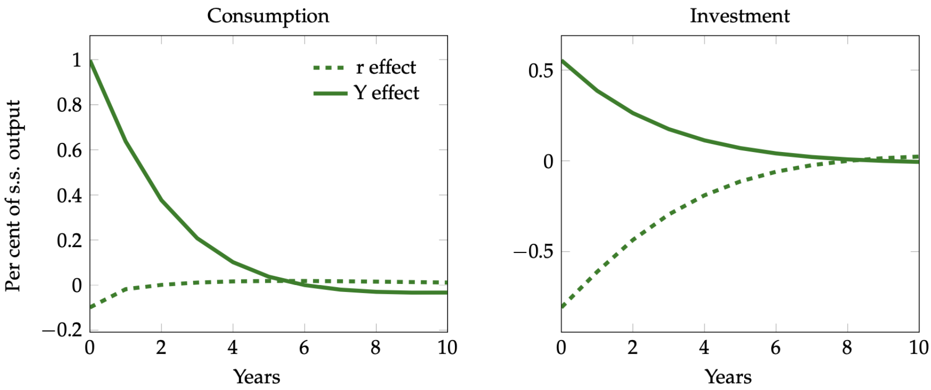
Multiplier in the Quantitative Model



- **Valerie Ramey:** multiplier for temporary deficit-financed spending is "probably between 0.8 and 1.5".

Decomposing the Responses

Figure 6: Decomposing the consumption and investment responses



Extensions and Other Shocks

- Generalization of the iKC allow to separate the effect of **public** and **private** deficit:

$$d\mathbf{Y} = \underbrace{d\mathbf{G} - d\mathbf{T}}_{\text{public deficits}} + \underbrace{(\mathbf{I} - \mathbf{M})d\mathbf{T} + \partial\mathbf{C}}_{\text{PE private deficits}} + \mathbf{M}d\mathbf{Y}$$

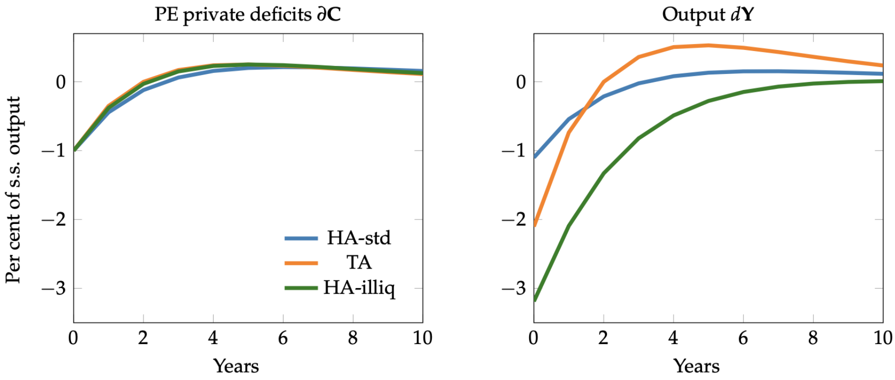
  where $\partial\mathbf{C}$ is the direct consumption effect of a shock to HH, prior to any GE feedback.

- The **PE private deficits** combines:
  - Net HH spending $(\mathbf{I} - \mathbf{M})d\mathbf{T}$ from change in taxes;
  - Direct effect $\partial\mathbf{C}$ of the shock on HH consumption.

- Illustrate with two examples: **deleveraging shock** and **lump-sum financed government spending**.

Deleveraging Shock

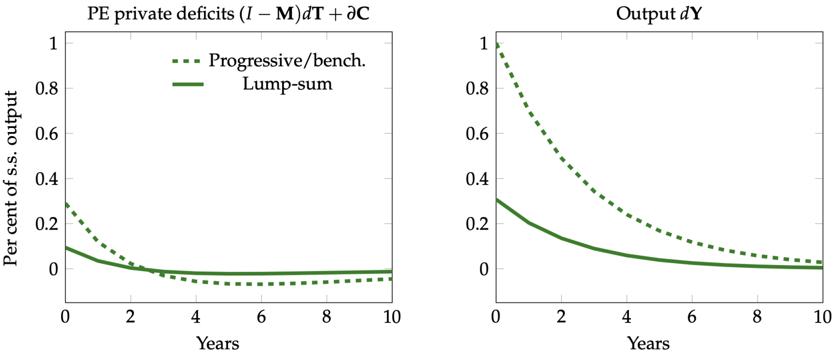
Figure 8: The effects of deleveraging shocks.



- **Deleveraging Shock**: Tightening of borrowing constraint $\underline{a}$.
- The deleveraging shock acts as a reduction of the private deficit and is captured by $\partial \mathbf{C}$.

Fiscal Policy is Less powerful if Financed by Lump-sum Taxes

Figure 9: Comparing two ways to finance government spending: progressive vs. lump-sum taxation.



- Lower PE private deficits on impact under lump-sum ⇒ This taxation targets many constrained households who have little ability to smooth consumption.

Conclusion

- New set of moments captures the GE effects of fiscal policy: **iMPCs**.
- HA with low liquidity matches the iMPCs of the data.
- Balanced-budget fiscal policy is weak even without heterogeneity.
- Deficit-financed fiscal policy is powerful and may have high impact and cumulative multipliers!
- Novel results on distortionary taxation, active monetary policy and others!

Appendix

Sticky Wages: Unions

- Problem of union k:

$$\max_{\{W_{kt+\tau}\}} \sum_{\tau \geq 0} \beta^{t+\tau} \left(\int \{u(c_{it+\tau}) - v(n_{it+\tau})\} d\Psi_{it+\tau} - \frac{\psi}{2} \left(\frac{W_{kt+\tau}}{W_{kt+\tau-1}} - 1 \right)^2 \right)$$

subject to HH budget constraint and $N_{kt} = (W_{kt}/W_t)^{-\epsilon} N_t$ for all t.

- Using the fact that $\partial c_{it}/\partial W_{kt} = \partial z_{it}/\partial W_{kt}$ and $n_{it} \equiv \int_0^1 (W_{kt}/W_t)^{-\epsilon} N_t dk$, F.O.C implies

$$\int \left\{ \frac{\partial z_{it}}{\partial W_{kt}} u'(c_{it}) + \frac{\epsilon}{W_{kt}} \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} N_t v'(n_{it}) \right\} d\Psi_{it} \ldots$$

$$\ldots - \psi \left(\frac{W_{kt}}{W_{kt-1}} - 1 \right) \frac{1}{W_{kt-1}} + \beta\psi \left(\frac{W_{kt+1}}{W_{kt}} - 1 \right) \frac{W_{kt+1}}{W_{kt}} \frac{1}{W_{kt}} = 0$$

Sticky Wages: Unions

$$\psi\left(\frac{W_{kt}}{W_{kt-1}}-1\right)\frac{W_{kt}}{W_{kt-1}} = W_{kt}\int\left\{\frac{\partial z_{it}}{\partial W_{kt}}u'(c_{it}) + \frac{\epsilon}{W_{kt}}\left(\frac{W_{kt}}{W_t}\right)^{-\epsilon}N_t v'(n_{it})\right\}d\Psi_{it}\ldots$$
$$\ldots + \beta\psi\left(\frac{W_{kt+1}}{W_{kt}}-1\right)\frac{W_{kt+1}}{W_{kt}}$$

- Using $\pi_t^w = W_{kt}/W_{kt-1} - 1$ and $\partial z_{it}/\partial W_{kt}\cdot W_{kt} = \partial z_{it}/\partial n_{it}\cdot(1-\epsilon)N_{kt}$

$$\pi_t^w(1+\pi_t^w) = \frac{1}{\psi}W_{kt}\int\left\{\frac{\partial z_{it}}{\partial W_{kt}}u'(c_{it}) + \frac{\epsilon}{W_{kt}}N_{kt}v'(n_{it})\right\}d\Psi_{it} + \beta\pi_{t+1}^w(1+\pi_{t+1}^w)$$
$$\pi_t^w(1+\pi_t^w) = \frac{\epsilon}{\psi}\int N_{kt}\left\{v'(n_{it}) - \frac{(\epsilon-1)}{\epsilon}\frac{\partial z_{it}}{\partial n_{it}}u'(c_{it})\right\}d\Psi_{it} + \beta\pi_{t+1}^w(1+\pi_{t+1}^w)$$

and by symmetry in eq. $n_{it} = N_{kt} = N_t$ and $W_{kt} = W_t$. Back

All income is eventually spent

- Iterating the BC of an arbitrary agent forward (and imposing a NPG):

$$c_0 + a_0 = (1 + r_{-1})a_{-1} + z_0 \qquad \Rightarrow \qquad \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} c_t = (1 + r_{t-1})a_{-1} + \sum_{s=0}^{\infty} \frac{1}{(1+r)^t} z_t$$

- Aggregating all agents:

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} C_t(\{Z_s\}) = (1 + r_{t-1})a_{-1} + \sum_{s=0}^{\infty} \frac{1}{(1+r)^t} Z_t$$

- Taking the derivatives with respect to Z_s:

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} M_{t,s} = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \qquad \Leftrightarrow \qquad \sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^{t-s}} = 1. \quad \square$$

Back

Consumption Function of RA Model

- Since $\beta(1+r)=1$, the EE $c_t^{-\sigma}=\beta(1+r)c_{t+1}^{-\sigma} \Longrightarrow c_t=c_{t+1}=c_{t+s}$ for all $s=0,1\ldots$.
- From the budget constraint:

$$c_t + a_t = (1+r_{t-1})a_{t-1} + z_t \qquad \Rightarrow \qquad \beta c_t + \beta a_t = a_{t-1} + \beta z_t$$

- Iterating the BC at $t=0$ forward (and imposing a NPG):

$$c_0 + a_0 = (1+r_{-1})a_{-1} + z_0 \qquad \Rightarrow \qquad \sum_{s=0}^{\infty}\beta^s c_s = (1+r_{t-1})a_{-1} + \sum_{s=0}^{\infty}\beta^s z_s$$

- Since $c_0=c_s=C_t$, $z_s=Z_s$ and $(1-\beta)(1+r_{-1})=r_{-1}$:

$$\frac{C_t}{1-\beta} = \sum_{s=0}^{\infty}\beta^s z_s + (1+r_{t-1})a_{-1} \qquad \Rightarrow \qquad C_t = (1-\beta)\sum_{s=0}^{\infty}\beta^s Z_s + r a_{-1}. \quad \square$$

Back