The "Big Push" of International Trade: A Tale of Informality and Human Capital*

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Abstract

Informality is pervasive in many developing economies and is often accompanied by low investment in human capital. We study the effects of international trade through the lens of a general equilibrium model with heterogeneous firms and workers, featuring labor market frictions. Complementarity between workers' human capital investment and firms' hiring decisions can give rise to multiple equilibria. We show that trade can act as a coordinating device that triggers a Big Push, shifting the economy from a "bad" equilibrium, with low human capital and high informality, to a "good" equilibrium, where firms have stronger incentives to formalize and workers invest more in human capital. The novel mechanism we uncover involves effort in human capital investment as a key margin of adjustment to trade shocks.

JEL Codes: E24, F12, F16, J24, J46, O17

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1 Introduction

Under which conditions can trade trigger a Big Push? This question speaks to broader debates on the links between trade, development, and structural transformation. A common view is that investment in human capital plays a central role in structural transformation and growth (see, e.g., Lucas, 1988, Barro, 1991, Gennaioli et al., 2013, Porzio et al., 2022). Another leading view of economic development emphasizes the process of industrialization, highlighting the role of complementarities and coordination failures in technology adoption (see, e.g., Rosenstein-Rodan, 1943, Murphy et al., 1989, Ciccone, 2002). According to this view, public policies play a crucial role in coordinating the decisions of firms and workers. A separate line of research focuses on how domestic distortions in developing countries contribute to development gaps and shape the effects of international trade (see, e.g., Bai et al., 2024; Dix-Carneiro et al., 2026). Such distortions can give rise to informality, a widespread phenomenon in developing countries. Yet, how trade interacts with informality and human capital to generate–or prevent–a Big Push remains poorly understood.

This paper proposes a "Big Push via trade" mechanism: trade acts as a coordinating device that simultaneously incentivizes human capital investment and firm formalization. Because these decisions are complementary, trade can shift the economy across equilibria. Our main result is that trade can move an economy from a low-human-capital, high-informality equilibrium to a high-human-capital, formal equilibrium.

We develop a model with heterogeneous firms and workers in which formality and human capital choices are jointly determined. Complementarities between firms' formalization decisions and workers' human capital investment generate multiple equilibria. Embedding this environment in an open economy allows trade to affect both margins and, under specific conditions, coordinate the economy on the high-development equilibrium.

Our framework builds on three elements: informality, human capital, and Big Push complementarities. Informality—the part of the economy that evades taxes and does not comply with labor market regulations—(Dix-Carneiro et al., 2026) constitutes a large share of employment and GDP in developing countries and economies are unlikely to simply grow out of it (Belavadi, 2021). Empirical evidence shows that informality is an intra-industry phenomenon, that is, informal and formal firms compete for labor within the same industries (Ulyssea, 2018). It also establishes that informal firms are smaller, less productive, and employ less educated workers (e.g., La Porta and Shleifer, 2008, 2014), and that informality is strongly negatively correlated with development (see Ulyssea, 2025 for a comprehensive survey). As Breza and Kaur, 2025 emphasizes, structural transformation–a move from informal jobs in traditional sectors toward more formal jobs in modern sectors–requires changes in human capital levels, better functioning labor markets, and, often, targeted public policy interventions.

Human capital stands out as the key margin of adjustment: shifts toward more educated workers play a central role in reducing informality, as the Brazilian experience makes clear (Haanwinckel and Soares, 2021). Yet the transition from a high-informality, low-human-capital equilibrium to a better one is not automatic. Complementarities between workers' and firms' decisions can give rise to coordination failures and informality traps.

The notion of Big Push has proven relevant for understanding such traps. It dates back to Rosenstein-Rodan, 1943, 1961, was further formalized by Murphy et al., 1989, and has seen renewed interest in recent work (see Kreickemeier and Wrona, 2020 and Buera et al., 2026). The idea is historically linked with industrial policy and presupposes the existence of a minimum scale for economic development. In the presence of multiple equilibria, lack of coordination can lead to underdevelopment, requiring government intervention to coordinate the economy towards the good equilibrium. This perspective naturally raises the question of whether external forces–such as trade–may act as coordination devices.

In recent decades, many developing countries have progressively reduced trade barriers while implementing industrial policies as part of their development agendas. Building on this observation, we formalize a Big Push via trade mechanism, in which trade acts as a coordinating device that incentivizes workers to invest in human capital and firms to formalize. Complementarities between firms' and workers' decisions play a crucial role in our framework, as they can generate multiple equilibria. We characterize the conditions under which trade acts as an equilibrium-switching force, moving the economy from a low-human-capital, high-informality equilibrium to one characterized by higher human capital and greater formalization.

Our paper contributes to several strands of the literature. First, it connects to trade models of heterogeneous firms and workers in the presence of imperfect labor markets. The most closely related contribution is Helpman et al., 2010.

Moreover, it builds on recent work on informality. Relevant studies include both closed-economy frameworks (see Erosa et al., 2023; Ulyssea, 2018) and open-economy analyses (Bosch et al., 2012; Dix-Carneiro et al., 2026; Dix-Carneiro and Kovak, 2019; Goldberg and Pavcnik, 2003; Ponczek and Ulyssea, 2022), with the latter yielding partly conflicting results on the informality consequences of trade liberalization. In particular, Dix-Carneiro et al., 2026 develops an equilibrium trade model with firm dynamics and firm heterogeneity, formal and informal sectors, labor market frictions, and a rich institutional setting to account for general equilibrium effects. A key quantitative result is that trade liberalization leads to a net decline in informality.

This paper is importantly related to the Big Push literature, and particularly to Murphy et al., 1989 and Kreickemeier and Wrona, 2020. The latter shows that moving from autarky to free trade generates a pro-competitive effect from the entry of foreign firms, which makes the coordinated adoption of increasing-returns technologies–its notion of a Big Push–more difficult. However, they argue that in models where trade induces efficiency-enhancing reallocation across firms (as in Melitz, 2003), trade can instead facilitate a Big Push by expanding high-productivity firms and driving out low-productivity ones.

More broadly, our analysis relates to the literature on human capital and development, particularly work that emphasizes complementarities between workers' and firms' decisions. The paper most closely related to ours is Bonfiglioli and Gancia, 2019, which establishes the existence of multiple equilibria driven by the complementarity between workers' effort and firms' screening intensity in a closed economy.

A further connection lies with the recent trade and development literature. Specifically, Atkin et al., 2025 shows that by affecting a country's pattern of specialization—i.e., shifting production toward more complex goods—trade can accelerate capability growth and promote development.

Our work is also related to studies of the effects of trade liberalization on skills. In particular, Bustos, 2011 develops a model in which trade liberalization prompts the most productive firms (exporters) to adopt more skill-intensive technologies. Empirical evidence from Brazil's tariff reductions shows that the most productive Argentine firms upgrade their skill use, while the least productive firms downgrade. Salient contributions to the trade and education literature include Blanchard and Willmann, 2016, which develops a model of endogenous skill acquisition in which trade liberalization can polarize both employment and human capital investment, and Atkin, 2016, which provides empirical evidence that trade openness in Mexico induced shifts in individuals' schooling decisions.

The distinctive aspect of our paper is the focus on how human capital, informality, and trade interact. We show that trade is not only reallocative–the main lesson from Melitz, 2003–but, under certain conditions, also acts as an equilibrium-switching shock. We thus extend the paradigm of Bonfiglioli and Gancia, 2019, which explains why an economy can get stuck in a bad equilibrium, by showing how trade can facilitate the escape from that equilibrium by acting as a coordinating device.

To assess the quantitative relevance of the mechanism, we calibrate the model to an economy with features representative of Brazil: a large informal sector and significant labor market frictions. We do not attempt to identify the equilibrium regime occupied by Brazil at any particular point in time. Rather, the quantitative analysis characterizes the conditions under which a Big Push via trade may arise and illustrates the threshold-dependent nature of the mechanism. Brazil provides a natural calibration target given the availability of matched employer–employee data and detailed survey evidence on informal firms.

The remainder of the paper is structured as follows: Section 2 presents the model, derives the conditions under which multiple equilibria arise, and studies the impact of trade. Section 3 conducts a quantitative analysis of the model. Section 4 concludes.

2 The model

We propose a general equilibrium model of heterogeneous firms and workers that builds on Helpman et al., 2010, extending it to include two sectors: formal and informal. The key feature of the model is a complementarity between firms' screening decisions and workers' effort choices, which can generate multiple equilibria. Labor markets are characterized by search and matching frictions. Workers, whose ability is unknown, are randomly matched with firms either in the formal or in the informal sector. To isolate the core mechanism, we assume that formal firms can screen workers to select them on the basis of an imprecise signal of their ability, whereas informal firms cannot. As in Bonfiglioli and Gancia, 2019, workers can invest costly effort to improve their ability, making the ability distribution endogenous.

Our model features reduced-form costs which are intended to capture, in a parsimonious way, the many kinds of imperfections, regulatory burdens, and compliance costs associated with formal and informal employment. This modelling choice is deliberate: rather than providing an exhaustive description of labor market institutions, we aim to preserve sufficient tractability in order to highlight the key mechanism.[1]

We consider a world of two countries, home and foreign (denoted by a tilde), and begin with a closed-economy setup before turning to the open economy.

2.1 Preferences and demand

As in Helpman et al., 2010, preferences are given by a CES utility over a continuum of differentiated varieties indexed by j:

$$U = Q = \left[\int_{j \in J} q(j)^{\beta} dj\right]^{\frac{1}{\beta}}, \quad 0 < \beta < 1$$

where Q represents the sector's real consumption index, J is the set of varieties within the sector, $q(j)$ denotes the consumption of variety j, and β is the elasticity of substitution be-

[1] A richer treatment of labor market institutions, including minimum wages, firing costs and unemployment benefits, is likely to matter for transition dynamics and is left for future work.

tween varieties. The demand function can therefore be written as $q(j) = A^{1/(1-\beta)} p(j)^{-1/(1-\beta)}$, where A is the demand shifter for the sector and the equilibrium revenue is $r(j) = p(j)q(j) = Aq(j)^{\beta}$.[2] We choose the aggregate consumption index as the numeraire ($P = 1$), so $E = Q$ and $A = Q^{1-\beta}$.

2.2 Firms' decisions: timing

The timing of firms' decisions unfolds as follows. First, firms observe their productivity θ, which is independently distributed and drawn from a Pareto distribution $G_{\theta} = 1 - \theta^{-z}$, with support on $[1, \infty)$ and shape parameter $z > 1$, after incurring a sunk cost f_e in units of the numeraire. Second, conditional on the productivity draw and the sector-specific fixed and variable costs, the firm decides whether to operate in the informal or formal sector.[3] Third, firms are randomly matched with workers. Formal firms engage in worker screening, whereas informal firms hire without screening. Fourth, firms and hired workers bargain over wages.

2.3 Formal sector

Output in the formal sector is given by the firm's productivity θ, the measure of workers hired h_f, and their average ability $\bar{a}$:

$$y_f = \theta h_f^{\gamma} \bar{a}, \quad \gamma \in (0, 1) \tag{1}$$

This production technology captures the complementarity between firm productivity and worker ability, which is central to the model's mechanism.[4] Workers' ability is match-specific and unobserved, but it is assumed to be independently distributed and drawn from a Pareto distribution $G_a(a) = 1 - a^{-k}$, with support on $[1, \infty)$ and shape parameter $k > 1$.[5]

Firms are subject to a fixed cost c_f, capturing the firms' registration with the tax authorities, and to a variable sales tax $\tau_f \in (0, 1)$.[6]

[2] $A = E^{1-\beta} P^{\beta}$, where $E = PQ$ is total expenditure and $P \equiv [\int p_j^{-\beta/(1-\beta)} dj]^{-(1-\beta)/\beta}$ is the sector's ideal price index.

[3] Two remarks are here in order. First, we consider only the extensive margin of informality, namely, firms not registering their business. While modeling the intensive margin, that is, formal firms hiring informal workers off the books, is empirically relevant (see, e.g. Ulyssea, 2018), we abstract from it for tractability. Second, the initial formalization decision is permanent, that is, no transition between the informal and formal sectors is allowed after entry.

[4] See, for instance, the O-ring and the span-of-control literature.

[5] In Appendix L, we discuss the implications of alternative distributions.

[6] Tax proceeds are rebated in a lump-sum manner to workers.

The labor market features search and matching frictions. A firm incurs a search cost of $b_f n_f$ units of the numeraire to randomly match with n_f workers, where b_f is endogenously determined by the tightness of the labor market (as described in 2.5). In addition, firms can pay a cost of ca_c^δ/δ units of the numeraire, with $c > 0$ and $\delta > k$, to screen workers, whose ability is unknown.[7] Firms use this screening technology to hire workers whose match-specific ability is above a chosen threshold a_c, but they cannot observe workers' precise ability: each worker is treated as if they possess an ability level corresponding to the expected value of the distribution $\bar{a}$. Workers whose realized match-specific ability falls below a_c are not hired and return to the unemployment pool. Under the assumption that worker ability follows a Pareto distribution with shape parameter k, the measure of hired workers is given by $h_f = n_f(1 - G(a_c)) = n_f a_c^{-k}$, whereas the average ability among hired workers is $\bar{a} = \mathbb{E}[a \mid a \geq a_c] = ka_c/(k-1)$. Output can therefore be written as $y_f(\theta) = k\theta n_f^\gamma a_c^{1-\gamma k}/(k-1)$, with $\gamma < 1/k$.

After the random productivity draw, a firm chooses whether or not to produce, how many workers n to match with (or sample) and its screening threshold a_c. Finally, firms and hired workers engage in bilateral wage bargaining *à la* Stole and Zwiebel, 1996a,b: each worker receives the share $\beta\gamma/(1+\beta\gamma)$ of average revenue per worker after taxes, whereas each firm receives the residual share $1/(1+\beta\gamma)$.

The θ-firm profit maximization problem can therefore be written as:

$$\pi_f(\theta) = \max_{\substack{n_f \geq 0 \\ a_c \geq 1}} \left\{ \frac{(1-\tau_f) r_f(\theta)}{1+\beta\gamma} - b_f n_f - \frac{c}{\delta}(a_c)^\delta - c_f \right\}.$$

The firm's first order conditions are

$$\frac{(1-\tau_f)\beta\gamma}{1+\beta\gamma} r_f(\theta) = b_f n_f(\theta) \tag{2}$$

$$\frac{(1-\tau_f)\beta(1-\gamma k)}{1+\beta\gamma} r_f(\theta) = c a_c(\theta)^\delta. \tag{3}$$

From the division of revenue in the bargaining game, wages are a constant share of after-tax revenue:

$$w_f(\theta) = \frac{\beta\gamma}{1+\beta\gamma} \frac{(1-\tau_f) r_f(\theta)}{h_f(\theta)} = b_f \frac{n_f(\theta)}{h_f(\theta)} = b_f a_c(\theta)^k. \tag{4}$$

These optimality conditions imply that more productive formal firms, that is, firms with

[7] Information frictions like missing markets for skill certification are a salient feature of labor markets in developing economies (see Breza and Kaur, 2025).

larger revenue, sample (and hire) more workers, are more selective, and pay higher wages, in a way consistent with the stylized facts. Unobserved ability and screening induce *imperfect* assortative matching: higher-ability workers tend to be employed by more productive firms. Moreover, the expected wage conditional on being sampled is the same across all firms:

$$\frac{w_f(\theta)h_f(\theta)}{n_f(\theta)} = b_f \tag{5}$$

so that workers have no incentive to direct search across firms.

2.4 Informal sector

Firms operating in the informal sector randomly match with and hire n_i workers after paying the search cost $b_i n_i$ (in units of the numeraire). Since no screening takes place, workers' ability does not enter the production technology:[8]

$$y_i = \theta n_i^{\gamma}. \tag{6}$$

Hired workers come from the untruncated Pareto distribution, so their average ability is the unconditional mean $k/(k-1)$. Moreover, firms pay a fixed cost $c_i < c_f$ and are subject to an informality penalty, that is, a size-dependent cost that accounts for the risk of being detected by the local authorities and the eventual punishment: $r_i(\theta)(1-\theta^{-\tau_i\beta})$ with $\tau_i \in (0,1)$.[9] As in the formal sector, a firm incurs a search cost $b_i n_i$ (in units of the numeraire) to randomly match with n_i workers, where b_i is endogenously determined by the tightness of the labor market (as described in 2.5).

The firm revenue is given by $r_i(\theta) = Ay_i(\theta)^{\beta} = A(\theta n_i^{\gamma})^{\beta}$.

The θ-firm solves the profit maximization problem

$$\pi_i(\theta) = \max_{n_i > 0} \left\{ \frac{\theta^{-\tau_i\beta}}{1+\beta\gamma} A(\theta n_i^{\gamma})^{\beta} - b_i n_i - c_i \right\}.$$

These first order conditions imply that, in contrast to the formal sector, there is no comple-

[8]This modeling choice is motivated by empirical evidence showing that, conditional on firm size, informal firms tend to hire less-skilled workers. One interpretation is that informal firms may be more intensive in raw labor and may produce lower-quality varieties (see Breza and Kaur, 2025 for a recent review of labor markets in developing countries). A further remark is in order. This model abstracts from physical capital, although we acknowledge its relevance. In practice, many firms in developing countries may remain informal due to limited access to capital or elevated borrowing costs (see Catão et al., 2009; Coşar et al., 2016). Such constraints may inhibit firms' expansion and, in turn, weaken workers' incentives to invest in human capital, as formal jobs may not materialize.

[9]The costs of informality are rebated in a lump-sum manner to workers.

mentarity between firm productivity and worker ability:

$$\frac{\theta^{-\tau_i \beta}\beta\gamma}{1+\beta\gamma} r_i(\theta) = b_i n_i(\theta) \tag{7}$$

$$w_i = \frac{\beta\gamma}{1+\beta\gamma} \frac{\theta^{-\tau_i\beta} r_i(\theta)}{n_i(\theta)} = b_i \tag{8}$$

that is, all informal workers receive the same wage irrespective of the firm's productivity, reflecting the absence of sorting and selection.

2.5 Search costs

Following Helpman et al., 2010 and the standard Diamond-Mortensen-Pissarides approach, we assume that search costs b_f and b_i are increasing in labor market tightness, x_f and x_i, respectively. We assume common search costs across sectors:

$$b_f = \alpha_0 x_f^{\alpha_1}, \tag{9}$$

$$b_i = \alpha_0 x_i^{\alpha_1}, \quad \alpha_0 > 0, \alpha_1 > 0 \tag{10}$$

where $x_s \equiv N_s/L_s$, for $s = f, i$.

The corresponding expected payoffs are:

$$\omega_f = x_f b_f \tag{11}$$

$$\omega_i = x_i b_i. \tag{12}$$

Combining equations (9) and (11) (and analogously (10) and (12)) yields the search cost and the labor market tightness for a given value of expected income:

$$b_f = \alpha_0^{\frac{1}{1+\alpha_1}} \omega_f^{\frac{\alpha_1}{1+\alpha_1}} \quad \text{and} \quad x_f = \left(\frac{\omega_f}{\alpha_0}\right)^{\frac{1}{1+\alpha_1}} \tag{13}$$

$$b_i = \alpha_0^{\frac{1}{1+\alpha_1}} \omega_i^{\frac{\alpha_1}{1+\alpha_1}} \quad \text{and} \quad x_i = \left(\frac{\omega_i}{\alpha_0}\right)^{\frac{1}{1+\alpha_1}} \tag{14}$$

where we choose parameter values for which $\alpha_0 > \omega_f$ and $\alpha_0 > \omega_i$ so that $0 < x_s < 1$ for $s = f, i$.

2.6 Profits and cutoffs

The profit functions of the formal firm and informal firm with productivity θ are given by

$$\pi_f(\theta) = \frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)}b_f^{-\frac{\beta\gamma}{\Gamma}}c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}}Q^{\frac{1-\beta}{\Gamma}}\theta^{\frac{\beta}{\Gamma}} - c_f \quad (15)$$

$$\pi_i(\theta) = \frac{1-\beta\gamma}{1+\beta\gamma}\phi_3^{\beta\gamma}b_i^{-\frac{\beta\gamma}{1-\beta\gamma}}Q^{\frac{1-\beta}{1-\beta\gamma}}\theta^{\frac{\beta(1-\tau_i)}{1-\beta\gamma}} - c_i \quad (16)$$

respectively, where Γ, ϕ_1, ϕ_2 and ϕ_3 are defined in Appendix B.

Given that $c_f > c_i$, π_f cuts π_i from below once.[10]

We relegate the formalization decision to Appendix C and focus here on intuition. Figure I plots the profit functions and the entry cutoffs θ_i^* and θ_f^* for the informal and formal sector, respectively.

The cutoff $\theta_i^* = \left(\frac{c_i}{\frac{1-\beta\gamma}{1+\beta\gamma}\phi_3^{\beta\gamma}b_i^{-\frac{\beta\gamma}{1-\beta\gamma}}Q^{\frac{1-\beta}{1-\beta\gamma}}}\right)^{\frac{1-\beta\gamma}{\beta(1-\tau_i)}}$ represents the break-even point, so that firms drawing $\theta < \theta_i^*$ exit immediately, otherwise they would make negative profits; firms with $\theta_i^* \le \theta < \theta_f^*$ enter the informal sector and firms with $\theta \ge \theta_f^*$ enter the formal sector.[11] Note that, as $\tau_f \to 1$, $\theta_f^* \to +\infty$ and, as $\tau_i \to 1$, $\theta_i^* \to +\infty$.

2.7 Workers' choices

Workers are *ex-ante* homogeneous and choose (i) whether to enter the formal or the informal sector and (ii) whether to exert costly effort to acquire formal education. These two decisions are linked, as effort affects the distribution of ability and therefore the returns to employment in the formal sector. This is a static, one-shot game with within-period sequencing of decisions. In equilibrium, a measure $L_f \le \bar{L}$ of workers choose the formal sector, which requires formal education at cost ε (in units of the numeraire). The remaining $L_i = \bar{L} - L_f$ workers choose the informal sector, where formal education is not required. Following Bonfiglioli and Gancia, 2019, ability is determined by two components: (i) a match-specific ability draw $\tilde{a}$, which is exponentially distributed, $Pr[\tilde{a} > x] = \exp(-x)$; (ii) an effort cost of η units of the numeraire.[12] Effort is modeled as a binary choice and denoted

[10] The concavity of π_i is straightforward using variable substitution: $\theta^{\frac{\beta}{\Gamma}} = t$ and $\theta^{\frac{\beta(1-\tau_i)}{1-\beta\gamma}} = \theta^{\frac{\beta}{\Gamma}\frac{\Gamma(1-\tau_i)}{1-\beta\gamma}} = t^m$, $0 < m < 1$, as $0 < \tau_i < 1$ and $\Gamma < 1-\beta\gamma$ given $0 < \gamma k < 1$.

[11] The cutoff θ_f^* does not, in general, admit a closed-form solution and is therefore computed numerically. The solution procedure is described in Appendix M.

[12] We interpret η as a deep parameter capturing non-pecuniary costs of effort, including psychological disutility and intrinsic motivation. In contrast, ε captures education and training costs, which are directly

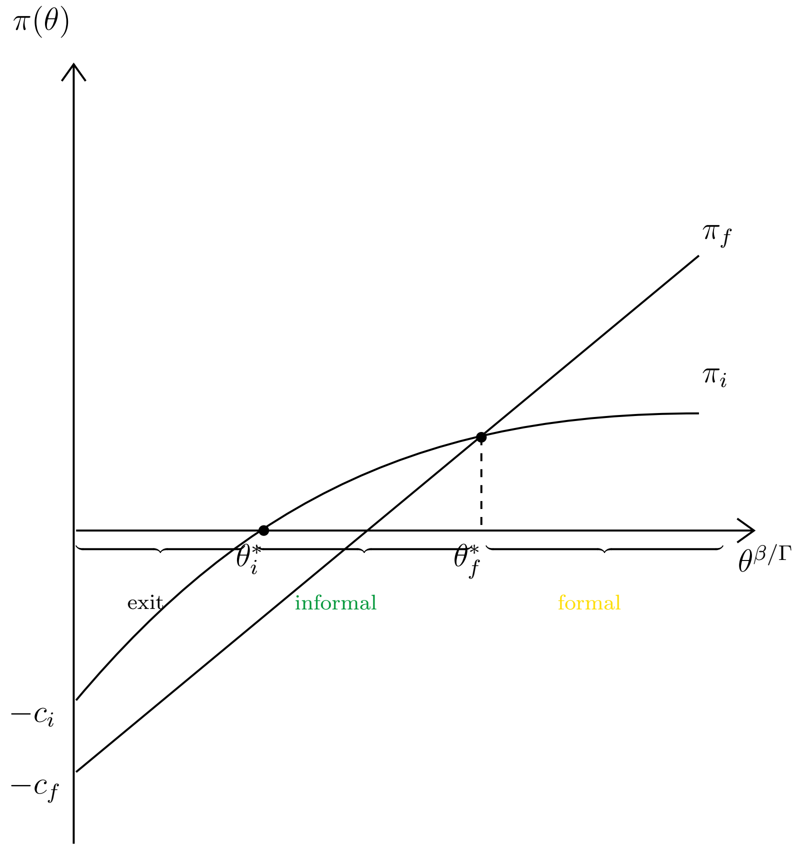

Figure I: Profits in the informal and formal sector as a function of productivity and respective entry cutoffs.

by the indicator function $\mathbb{I}_\eta \in \{0, 1\}$: exerting high effort ($\mathbb{I}_\eta = 1$) increases ability, whereas low effort ($\mathbb{I}_\eta = 0$) leaves it unchanged.[13] Ability is thus endogenously determined:

$$\ln a = \tilde{a}/k \quad \text{where } k = \begin{cases} k_0 < \frac{1}{\gamma} & \text{if } \mathbb{I}_\eta = 0 \\ k_1 < k_0 & \text{if } \mathbb{I}_\eta = 1 \end{cases}$$

which is Pareto distributed with shape parameter k, as is evident from $\Pr[a > x] = \Pr[\tilde{a} > k \ln x] = x^{-k}$; importantly, mean and variance are higher with high effort:

$$\mathbb{E}[\ln a | \mathbb{I}_\eta = 1] = (k_1)^{-1} > (k_0)^{-1} = \mathbb{E}[\ln a | \mathbb{I}_\eta = 0]$$
$$\mathbb{V}[\ln a | \mathbb{I}_\eta = 1] = (k_1)^{-2} > (k_0)^{-2} = \mathbb{V}[\ln a | \mathbb{I}_\eta = 0]$$

policy-relevant and can be affected by subsidies or schooling interventions.

[13] In Appendix K, we provide intuition about the implications of continuous effort choice.

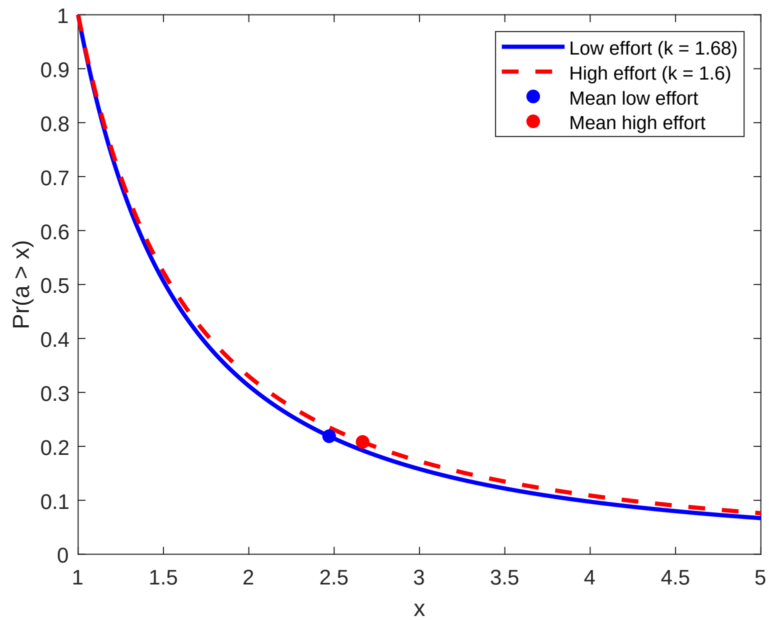

Figure II: Tail functions of ability distributions

where $\mathbb{E}$ is the expectation operator and $\mathbb{V}$ denotes the variance. As shown in Figure II, the probability that a worker's ability a is above a certain threshold x is higher with high effort (red dashed line) than with low effort (blue solid line).

The within-period sequencing of the workers' choices is illustrated in Figure III. First, workers select into either the informal or the formal sector. Second, those who choose the formal sector decide whether to exert low or high effort, leading to different ability distributions. Ability is match-specific and not observable by firms ex ante. Informal firms are assumed to know the underlying distribution of ability, but this information does not influence their hiring decisions, as all workers are observationally equivalent from their perspective.

In equilibrium, the following indifference condition must hold:

$$\omega_i = \omega_f - \varepsilon - \mathbb{I}_\eta \eta. \tag{17}$$

More specifically, the expected payoff in the informal sector, $\omega_i = b_i N_i / L_i \equiv w_i x_i$, that is, the expected wage conditional on being sampled times the probability of being sampled, must equalize the expected payoff in the formal sector, $\omega_f = b_f N_f / L_f \equiv b_f x_f$, less the education and effort costs. In other words, workers must be indifferent between entering the informal sector (i.e. the left hand side) or acquiring costly education to search for a job in the formal sector (i.e. the right hand side).

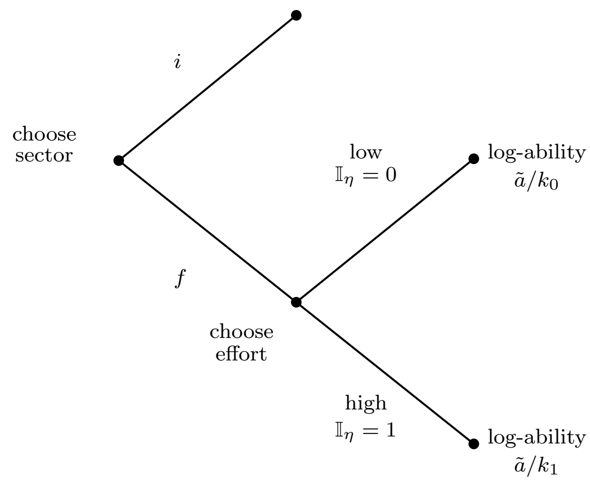

Figure III: Diagram of intra-period choices

We can obtain a relationship for the labor market tightness in the two sectors by combining (9), (10), and (17):

$$x_f = \left(\frac{\alpha_0 x_i^{1+\alpha_1} + \varepsilon + \mathbb{I}_\eta \eta}{\alpha_0} \right)^{\frac{1}{1+\alpha_1}}. \tag{18}$$

All else equal, the probability of matching with a formal firm is higher when effort is high.

To better understand how workers sort across sectors, it is helpful to look at the following equation, which links the sectoral labor ratio to the relative worker payoffs and to aggregate after-tax revenues:

$$\frac{L_f}{L_i} = \left(\frac{\omega_i + \varepsilon + \mathbb{I}_\eta \eta}{\omega_i} \right)^{-1} \frac{(1-\tau_f) \int_{\theta_f^*}^{\infty} r_f(\theta) dG(\theta)}{\int_{\theta_i^*}^{\theta_f^*} \theta^{-\tau_i \beta} r_i(\theta) dG(\theta)}. \tag{19}$$

The relative mass of job seekers in the formal sector is the product of two components: the (inverse of the) relative payoff; and the relative revenue after taxes. This condition is derived by imposing the equality of the expected wage and total labor payments in both sectors:

$$L_f(\omega_i + \varepsilon + \mathbb{I}_\eta \eta) = M_f(1-\tau_f) \frac{\beta\gamma}{1+\beta\gamma} \int_{\theta_f^*}^{\infty} r_f(\theta) dG(\theta) \tag{20}$$

$$L_i \omega_i = M_i \frac{\beta\gamma}{1+\beta\gamma} \int_{\theta_i^*}^{\theta_f^*} \theta^{-\tau_i \beta} r_i(\theta) dG(\theta) \tag{21}$$

where $M_f = M_i = M$ is the mass of formal and informal firms derived from the following market clearing conditions:

$$E = M\left(\int_{\theta_i^*}^{\theta_f^*} r_i(\theta)\, dG(\theta) + \int_{\theta_f^*}^{\infty} r_f(\theta)\, dG(\theta)\right) \tag{22}$$

$$E = Q \tag{23}$$

$$\frac{\beta\gamma}{1+\beta\gamma}Q - L_f(\varepsilon + \mathbb{I}_\eta \eta) = \omega_i \bar{L} + \frac{\beta\gamma}{1+\beta\gamma}T^a \tag{24}$$

where $T^a = M\left(\int_{\theta_f^*}^{\infty} \tau_f r_f(\theta) dG(\theta) + \int_{\theta_i^*}^{\theta_f^*} r_i(\theta)(1-\theta^{-\tau_i\beta})dG(\theta)\right)$ is aggregate tax revenue. The derivation of equation 24 is provided in Appendix F.

2.8 Multiple equilibria

The interaction between firms' screening decisions and workers' effort choices generates a coordination problem that can give rise to multiple equilibria. Bonfiglioli and Gancia, 2019 establish the existence of two pure-strategy equilibria sustained by the complementarity between workers' effort and firms' screening intensity in a closed-economy setup with two homogeneous goods.[14] We extend that result by including horizontally differentiated varieties. Let us denote with θ_0^* and θ_1^* the formal-sector cutoffs when $k = k_0$ and $k = k_1$ respectively and define $\Delta k \equiv k_0 - k_1 > 0$.

Proposition 1. *Given $k_1 < k_0 < 1/\gamma$,*

$$\frac{Q_1^{(1-\beta)/\Gamma_1}}{Q_0^{(1-\beta)/\Gamma_0}} > \frac{b_{f,1}^{\beta\gamma/\Gamma_1}}{b_{f,0}^{\beta\gamma/\Gamma_0}} c^{(1-\beta\gamma)\left(\frac{1}{\Gamma_1}-\frac{1}{\Gamma_0}\right)}, \tag{25}$$

and

$$\underbrace{\frac{\delta z\Gamma_0 a_c(\theta_0^*, k_0)^{\Delta k}}{\delta z\Gamma_0 - \beta\Delta k}}_{\equiv A_{min}} < \frac{\omega_i + \varepsilon + \eta}{\omega_i + \varepsilon} < \underbrace{\left(1 + \frac{\beta\Delta k}{\delta z\Gamma_1}\right) a_c(\theta_1^*, k_1)^{\Delta k}}_{\equiv A_{max}}, \tag{26}$$

there exist only two pure-strategy equilibria sustained by different workers' beliefs on firms' screening decisions. In the good equilibrium all job seekers in the formal sector exert high effort ($\mathbb{I}_\eta = 1$) and formal firms with productivity θ screen workers at a cutoff ability $a_c(\theta, k_1)$, whereas in the bad equilibrium, investment in effort is low ($\mathbb{I}_\eta = 0$) and firms screen workers at a lower cutoff for any productivity level, $a_c(\theta, k_0) < a_c(\theta, k_1)$.

Proof. See Appendix D. □

[14] Bonfiglioli and Gancia, 2019 discuss how different beliefs on the value of effort for finding good jobs (e.g. hard work vs luck and/or personal connections) drive different countries to one equilibrium or the other.

This result can be schematized as in Figure IV. The respective payoffs–expected wages and profits–are not reported. Intuitively, high effort and screening (good equilibrium) lead to higher equilibrium ability and a stronger sorting pattern; low effort and low screening (bad equilibrium) lead to lower equilibrium ability and a weaker sorting pattern.

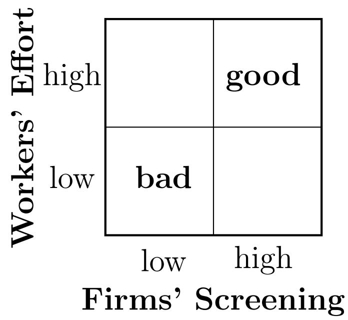

Figure IV: 2x2 payoff matrix with multiple equilibria

Multiplicity arises because the returns to screening depend on effort, which, in turn, depends on expected screening. Formal entry therefore depends on productivity and expectations: the cutoff productivity level for formal entry is endogenous and determined by the ability distribution, characterized by the shape parameter k. Firms below this cutoff cannot profitably operate formally. To better understand the source of multiple equilibria, consider first the optimal behavior of firms. The value of screening is proportional to the degree of heterogeneity in worker ability. Firms know the distribution parameter k and optimally choose their screening threshold. When workers exert high effort, the ability distribution becomes more dispersed, raising the returns to screening and inducing firms to screen more intensively. Conversely, when effort is low, the reduced heterogeneity diminishes the incentive to screen. This complementarity can generate a coordination failure, with the economy potentially trapped in a low-effort, high-informality equilibrium because no agent has a unilateral incentive to deviate.

Human capital and screening are jointly strategic margins in the model: workers choose effort based on expected returns in the formal sector, while firms choose screening intensity based on the expected ability distribution. This interaction generates the coordination problem underlying the Big Push.

The switch from bad to good then requires a force that raises the firms' incentive to screen, which in turn can make the choice of high effort self sustaining. This is because exerting low effort when firms are screening at high intensity results in a lower expected wage, with the penalty for low effort increasing in the screening threshold:

$$\underbrace{\mathbb{E}[w_f(k_0, a_c(k_1))]}_{\substack{\text{low-effort} \\ \text{high-screening} \\ \text{expected wage}}} = \underbrace{\frac{\delta z \Gamma_1 a_c(\theta_1^*, k_1)^{-\Delta k}}{\delta z \Gamma_1 + \beta \Delta k}}_{\text{punishment for low effort}} \underbrace{b_f \frac{N_{f,1}}{L_{f,1}}}_{\substack{\text{high-effort} \\ \text{high-screening} \\ \text{expected wage}}} < b_f \frac{N_{f,1}}{L_{f,1}} = \mathbb{E}[w_f(k_1, a_c(k_1))].$$

Corollary 1. *Let $R(a_c)$ denote the expected return to effort conditional on firms' screening intensity a_c. Under conditions (25) and (26), the bad equilibrium is unsustainable–and the good equilibrium is therefore the unique outcome–if and only if*

$$R(a_c(k_0)) \equiv \mathbb{E}[w_f(k_1) - w_f(k_0) | a_c(k_0)] > \eta. \tag{27}$$

Proof. See Appendix E. □

The good (bad) equilibrium therefore disappears if the effort cost is too high (low).[15] Graphically, one can visualize the multiplicity region in Figure V: the blue region represents

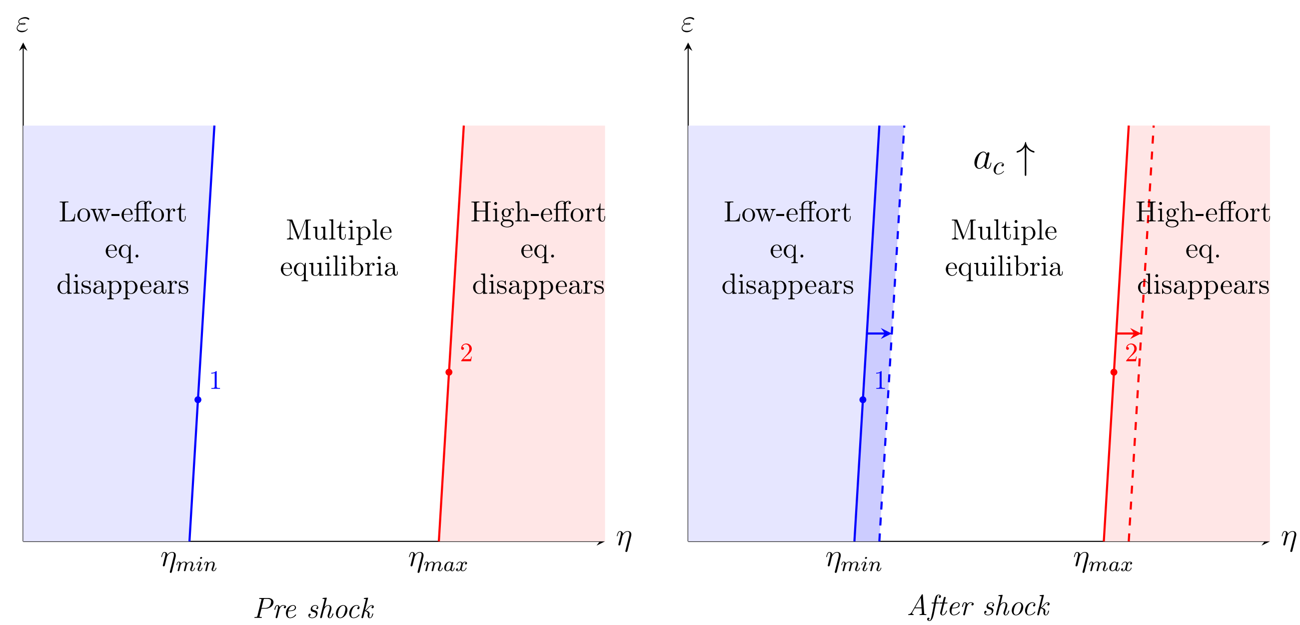

Figure V: Multiplicity region in the $(\varepsilon - \eta)$-space

the region in the $(\varepsilon - \eta)$-space where the low-effort (or bad) equilibrium is not sustainable, the red region is the region where the high-effort (or good) equilibrium is not sustainable

[15]From 26, one can pin down the combinations of ε and η such that the low-effort equilibrium disappears, i.e., when $\eta \leq \eta_{min} \equiv (\omega_i + \varepsilon)\left(\frac{\delta z \Gamma_0 a_c(\theta_0^*, k_0)^{\Delta k}}{\delta z \Gamma_0 - \beta \Delta k} - 1\right)$ and the high-effort equilibrium disappears, i.e., when $\eta \geq \eta_{max} \equiv (\omega_i + \varepsilon)\left(\left(1 + \frac{\beta \Delta k}{\delta z \Gamma_1}\right) a_c(\theta_1^*, k_1)^{\Delta k} - 1\right)$.

and the white region is the multiple equilibria region, that is, where both equilibria are sustainable. Suppose that the economy is initially in the low-effort equilibrium within the multiple equilibria region. Consider the left panel first: at point 1, the economy is characterized by a low effort cost, $\eta = \eta_{min}$. Even a small increase in the screening intensity (i.e., an increase in the screening threshold a_c), depicted in the right panel, eliminates the low-effort equilibrium by expanding the blue region. Workers find it profitable to deviate from the low-effort equilibrium and choose high effort. Suppose instead that the economy is still in the low-effort equilibrium but initially in point 2, representing a high effort cost, $\eta = \eta_{max}$. The same shock keeps the economy in the multiple equilibria region. A much larger shock to the screening intensity is now needed to make the switch to high effort.

We argue that trade can be a coordinating force that triggers the switch from the bad to the good equilibrium. However, before we turn to the analysis of the open economy, let us analyze the impact of alternative policy interventions targeting informality and formality costs: (i) a reduction in *variable* formality costs, i.e., lower τ_f; (ii) a reduction in *fixed* formality costs, i.e., lower c_f; and (iii) tighter enforcement on informal firms (e.g. more government auditing), i.e., higher τ_i.

The direct effect of policy (i) is to raise profits of formal firms: as figure I makes clear, π_f rotates upwards and the cutoff θ_f^* declines. The negative general equilibrium effect, that is, higher equilibrium wages in the whole economy, hurts both the formal and informal sector: the cutoffs θ_i^* and θ_f^* increase, so the least productive firms in both sectors are forced to exit.

Policy (ii) also raises profits of formal firms, although the direct positive effect is stronger for low-productivity formal firms, as fixed costs represent a larger fraction of their value. The negative wage effect again eliminates the least productive informal and formal firms.

Policy (iii) hurts directly informal firms: the cutoff θ_i^* rises and θ_f^* mechanically declines, thus inducing greater entry into the formal sector. The negative wage effect, on the other hand, increases both θ_i^* and θ_f^*, as before.

As we will see in section 2.10, the mechanisms at work under trade liberalization share some similarities with the three policy interventions discussed above. However, trade liberalization differs along an important dimension: its effects are concentrated in the right tail of the productivity distribution. The most productive firms benefit disproportionately and experience a large increase in revenues, which raises their incentives to screen workers. Higher dispersion in ability is critical for sustaining a high-effort equilibrium, as screening must increase sufficiently to change workers' effort choices. By disproportionately strengthening screening incentives for high-θ firms—those that benefit the most from higher ability dispersion—trade liberalization provides a natural coordination device for an equilibrium switch. That said, we do not view policies aimed at reducing formality costs as competing instruments. The core narrative of the paper is that trade, as part of broader globalization processes, can generate sharply different outcomes depending on an economy's position relative to the coordination threshold.

2.9 Open economy

We now open the economy to trade and assume that only formal firms can trade with the foreign economy (denoted by tilde). This is because firms have to formally register to undertake exporting procedures and also because it takes scale to become an exporter. Firms that export face a fixed cost c_x and a variable iceberg cost $\tau_x > 1$.[16] Total revenue is now the sum of domestic and exporting revenue:

$$\begin{aligned} r_f(\theta) &= r_d(\theta) + r_x(\theta) \\ &= Y(\theta)^{1-\beta} A y_f(\theta)^{\beta} \end{aligned}$$

where $Y(\theta)$ denotes the market access variable:

$$Y(\theta) = \begin{cases} 1 & \theta < \theta_x^* \\ Y_x > 1 & \theta \geq \theta_x^* \end{cases} \tag{28}$$

which is equal to 1 if the firm does not export and greater than 1 if it does export and Y_x is the intensive margin of trade openness:

$$Y_x \equiv 1 + \tau_x^{-\beta/(1-\beta)} \left(\frac{\tilde{A}}{A} \right)^{1/(1-\beta)} > 1.$$

The θ-formal firm now solves the following maximization problem:

$$\pi(\theta) = \max_{\substack{n_f \geq 0 \\ a_c \geq 1 \\ \mathbb{I}_x \in \{0,1\}}} \left\{ \frac{1-\tau_f}{1+\beta\gamma} Y(\theta)^{1-\beta} A \left[\frac{k}{k-1} \theta n_f^{\gamma} a_c^{1-\gamma k} \right]^{\beta} - b_f n_f - \frac{c}{\delta} (a_c)^{\delta} - \mathbb{I}_x c_x - c_f \right\}, \tag{29}$$

that is, besides choosing the number of matches and the screening threshold, it also chooses

[16] In this model, we only consider export-side liberalization. An extension accounting for import-side liberalization (see, for instance, Coşar et al., 2016; Dix-Carneiro et al., 2026) is left for future work. In a reduced form, import-side liberalization lowers the domestic demand shifter A, thereby reducing revenues and screening incentives for domestic producers, while at the same time increasing the intensive margin of trade openness, Y_x, strengthening screening incentives for the most productive exporters. Whether the import-side effects dominate or are dominated by the export channel is ultimately an empirical question.

its export status, $\mathbb{I}_x \in \{0,1\}$. The first order conditions are given by (2) – (4). The profit function can be written in a general form as follows:

$$\pi(\theta) = \frac{\Gamma(1-\tau_f)}{1+\beta\gamma} r_f(\theta) - \mathbb{I}_x(\theta)c_x - c_f$$

$$= \frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} [Y(\theta)^{1-\beta}\theta^{\beta}]^{\frac{1}{\Gamma}} - \mathbb{I}_x(\theta)c_x - c_f \tag{30}$$

$$= \begin{cases} \pi_f(\theta) & \text{if } \mathbb{I}_x(\theta) = 0 \\ \pi_x(\theta) & \text{if } \mathbb{I}_x(\theta) = 1 \end{cases} \tag{31}$$

The θ-firm profit function $\pi_x(\theta)$ therefore represents total profit from domestic production and exporting in the formal sector, while the profit function $\pi_f(\theta)$ denotes profit from domestic production only. As in the closed economy section, we relegate the formalization decision to Appendix H and focus here on intuition. Figure VI plots the profit functions as well as the cutoffs θ_i^*, θ_f^* and θ_x^* for informal firms, purely domestic formal firms and exporting formal firms, respectively. Firms with productivity $\theta_f^* \leq \theta < \theta_x^*$ become formal firms producing only in the domestic market, while firms with $\theta \geq \theta_x^*$ become exporters. Notice that the autarky scenario corresponds to infinite trade costs, $c_x = \infty$ and/or $\tau_x = \infty$, such that $\theta_x^* = \infty$.

2.10 The impact of trade: unpacking the mechanisms

We now turn to the effects of opening a closed economy to trade. Some of the mechanisms at work are already known, as they resemble those of Melitz-type models. However, the model we propose also incorporates a novel mechanism due to the presence of endogenous ability (human capital).

Proposition 2. *Suppose conditions (25) and (26) hold, and the economy is in the bad equilibrium under autarky. Then there exists a threshold trade cost $\bar{\tau}_x$ such that for any $\tau_x < \bar{\tau}_x$, trade opening satisfies condition (27), triggering a Big Push to the good equilibrium.*

Proof. See Appendix J. The three mechanisms below provide the intuition. □

Figure VII reports the changes in the cutoffs due to trade exposure. Although we refer to the mechanisms in sequential terms for expositional clarity, the model is static: the equilibrium is defined as a fixed point in which firms' screening decisions and workers' effort choices are mutually consistent.

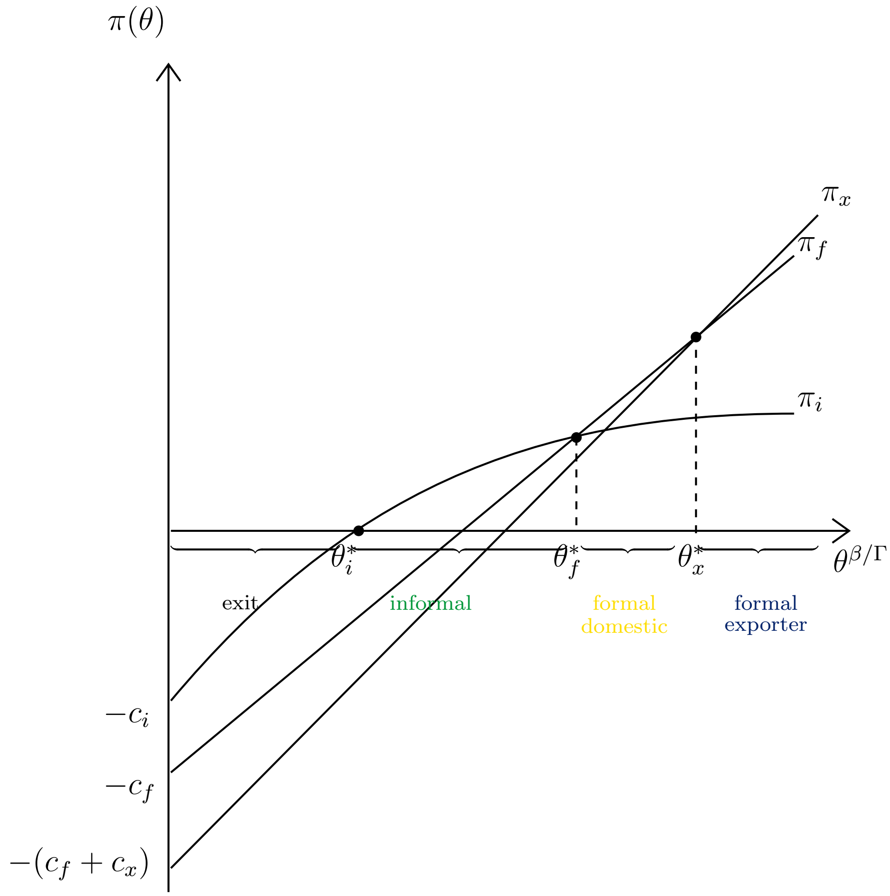

Figure VI: Profits of informal and formal (domestic and exporting) firms as a function of productivity and respective cutoffs.

First, opening to trade induces a selection process. Only formal firms with productivity above θ_x^* become exporters. Such firms experience a discrete jump in revenue; higher revenue, in turn, is shared with workers through higher wages. Higher revenue also translates into more resources devoted to screening and a higher screening threshold; this implies that some marginal workers are no longer hired by the same firm in the open-economy equilibrium.

Second, a domestic market selection effect takes place: access to the export market induces an increase in the domestic cutoffs θ_i^* and θ_f^* to $(\theta_i^*)'$ and $(\theta_f^*)'$, respectively; the cutoff θ_x^* increases as well, since formal exporters also operate in the domestic market. The least productive informal (formal) firms with productivity levels between θ_i^* and $(\theta_i^*)'$ (θ_f^* and $(\theta_f^*)'$), can no longer earn non-negative profits in the open economy equilibrium and are forced to exit. This occurs through a domestic channel. As the value of formality increases, incumbent exporters and new exporters bid up wages, thus forcing the least productive firms in both sectors to exit. This is the indirect general equilibrium effect on wages: profits in the formal domestic sector decline due to higher labor costs–as the most productive firms in the formal sector raise labor demand, equilibrium wages increase–, so that the least productive firms are pushed out and only the most productive exporters enjoy a net gain. Informal firms also experience a decline in profits, as they are now competing with higher wages and the least productive firms are wiped out: the informal sector contracts. This negative effect on domestic profits raises the cutoffs to θ_i^* and θ_f^*. The exit of the least productive firms also leads to an increase in average firm productivity and average worker ability. As a result, the effort-screening complementarity is strengthened.

Third, conditional on (27), when exporters screen intensively, workers choose high effort, and all firms optimally screen more. This is because workers' high effort makes the ability distribution more dispersed–i.e., k falls–, which raises firms' incentive to screen at a higher threshold. Given higher (average) ability, all formal firms optimally increase screening intensity, which raises revenues and profits and sustains the high-effort equilibrium. Note that purely domestic firms experience a decline in revenues due to higher wages; this would reduce their incentive to screen. However, the endogenous response of worker effort and the resulting shift in the ability distribution increase the expected returns to screening, so that domestic firms optimally screen more despite the negative wage effect. The new general equilibrium effect we highlight is driven by endogenous ability: high effort is a margin of adjustment to the trade shock. As a result of high effort, profits of formal firms increase and the formal cutoffs decline to $(\theta_f^*)''$ and $(\theta_x^*)''$. By the free entry condition, profits of informal firms decline, so the cutoff is raised to $(\theta_i^*)''$.

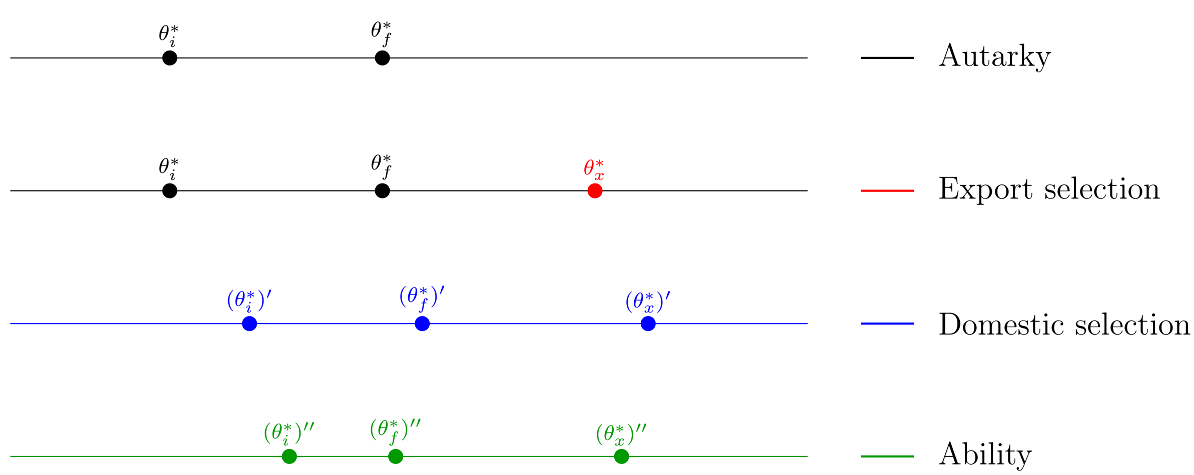

Figure VII: Cutoff levels under autarky (first line) and changes in the open economy due to selection effects (second and third line) and ability effect (fourth line).

Following Melitz, 2003, the previous analysis compared the equilibrium outcomes of an economy transitioning from autarky to an open economy. To bring the model to the data–we will focus on Brazil's trade liberalization–we now consider an increase in trade exposure (a

τ_x cut) and ask whether such a change generates similar effects.

First, trade liberalization benefits disproportionately the most productive firms, as before: greater access to the foreign market raises revenue and profits, but also wages and screening intensity. Graphically, a fall in τ_x makes the profit function π_x rotate to the left and the cutoff θ_x^* decline to a lower level, that is, the likelihood of exporting is now higher. This is the standard direct effect.

Second, by inducing a reallocation of resources toward high-wage exporters and an expansion of the formal sector, trade liberalization raises the cutoffs for the domestic market, both in the informal and in the formal sector. This is the indirect effect due to higher equilibrium wages discussed earlier.

Third, the most productive firms screen more intensively, which, conditional on (27), triggers a switch to high effort. Even in the absence of an equilibrium switch–that is, when the ability effect does not materialize–trade liberalization generates within-equilibrium reallocation: exporters expand, while the informal and domestic-oriented formal sectors contract.

In Appendix K, we analyze the implications of a continuous effort choice for our mechanisms.

Let us now examine the effects on aggregate informality.

Proposition 3. *The opening of trade has an ambiguous overall effect on aggregate informality. The net effect is captured by the change in the cutoff ratio θ_f^*/θ_i^* and cannot be signed analytically.*

Proof. Three counteracting forces operate simultaneously.

(i) Formality selection effect: the domestic selection effect raises θ_f^*, forcing the least productive formal firms to exit; the share of informal firms among all active firms rises.

(ii) Informality selection effect: the same selection mechanism raises θ_i^*, wiping out the least productive informal firms; the share of informal firms falls.

(iii) Ability effect: if trade triggers high effort, higher $\bar{a}$ raises formal-sector profitability, lowering θ_f^* relative to θ_i^* and further contracting the informal sector.

The net effect on informality depends on the relative magnitude of these forces, as summarized by the change in θ_f^*/θ_i^* relative to the pre-shock scenario. Since this ratio cannot be signed analytically, the overall effect is ambiguous. □

We therefore turn to a quantitative evaluation of these channels.

3 Quantitative exploration

In this section, we quantitatively assess the mechanisms analyzed in the theoretical section through counterfactual experiments.

3.1 Model calibration

We calibrate the model to a developing economy matching key moments of the Brazilian economy. Following Dix-Carneiro et al., 2026, the calibration uses 2003 data to discipline the model's structural parameters. We use Brazil as a laboratory to discipline parameter values, without claiming that the counterfactual replicates any specific historical episode.[17] The exercise characterizes the structural conditions under which a Big Push via trade is quantitatively feasible, and illustrates the threshold-dependent nature of the mechanism.

Table I lists the externally set parameters. We begin by assuming symmetric countries, $A = \tilde{A}$. We set $\beta = 0.8$, which corresponds to an elasticity of substitution within sectors of 5, a standard value in the trade literature. The death rate, equal to 10%, is taken from Erosa et al., 2023. Following Dix-Carneiro et al., 2026, the iceberg trade cost is set at $\tau_x = 2.4$, constructed using Head and Ries index and the 2003 World Input-Output Database (WIOD) applied to 2003 trade flows between Brazil and the rest of the world. The fixed cost of operating formally, c_f, is normalized to 1. The formal-sector sales tax is set to its statutory value, $\tau_f = 0.29$ (Ulyssea, 2018). The shape parameter of the productivity distribution z is taken from Bonfiglioli and Gancia, 2019 and set at 3.4. We set the low-effort shape parameter to $k_0 = 1.68$, following Bonfiglioli and Gancia, 2019, who calibrate it to Italy using IALS data (Cebreros, 2018). For high effort, we set $k_1 = 1.6$, corresponding to a reduction of about 5% relative to the low-effort case.[18] Following Bonfiglioli and Gancia, 2019 and Helpman et al., 2017, the return-to-hiring parameter is set to $\gamma = 0.5$, which satisfies the restriction $\gamma < 1/k$.

[17] Brazil's trade liberalization between 1986 and 1995 was largely unilateral and import-side, whereas our model considers export-side liberalization only.

[18] Bonfiglioli and Gancia, 2019 calibrate $k_1 = 1.27$ to the United States, implying a 24% reduction in k. Our choice of $k_1 = 1.6$ is more conservative and represents an achievable high-human-capital benchmark for a developing economy undergoing structural transformation.

Table I: Externally Set Parameters

Parameter		**Value**	**Source**
Elasticity of substitution	β	0.8	standard
Death rate	d	0.1	Erosa et al., 2023
Iceberg trade cost	τ_x	2.4	Dix-Carneiro et al., 2026
Formal-sector fixed cost	c_f	1	Normalization
Formal sales tax	τ_f	0.29	Ulyssea, 2018
Shape parameter productivity distr.	z	3.4	Bonfiglioli and Gancia, 2019
Shape parameter ability distr. (low effort)	k_0	1.68	Bonfiglioli and Gancia, 2019
Shape parameter ability distr. (high effort)	k_1	1.6	See text
Returns to hiring	γ	0.5	Bonfiglioli and Gancia, 2019; Helpman et al., 2017

Table II reports the parameters that are calibrated internally. The free-entry cost f_e is chosen so that the aggregate consumption index Q normalizes to 10. The search cost parameters α_0 and α_1 are set at the lowest value consistent with equilibrium restrictions. The curvature of the screening cost δ is set to match the employment-weighted standard deviation of log wages in the formal sector, such that $\sigma(\ln w_f) = \frac{k}{k+\delta\left(\frac{\Gamma z}{\beta}-1\right)} = 0.42$ (Helpman et al., 2017); this yields $\delta = 1.848$, satisfying $\delta > k$. We set the education cost ε to match exactly the conditional formal-informal log wage gap in Brazil (Ulyssea, 2018's estimate using PNAD data is 0.2864 after controlling for workers' skill, gender, seven-digit industry dummies, age, and age squared). This yields $\varepsilon = 0.46$, which is close to the value chosen by Bonfiglioli and Gancia, 2019. We set the effort cost η to its highest admissible value consistent with equilibrium multiplicity, so as to maximize the cost of switching to the high-effort equilibrium. We then contrast this benchmark with the lowest admissible value of η, which minimizes such cost. The resulting values are $\eta_{max} = 0.088$ and $\eta_{min} = 0.044$, respectively.[19]

We calibrate the remaining four parameters $\Theta = \{c, c_i, \tau_i, c_x\}$ using the Simulated Method of Moments (SMM) in an overidentified system that targets five empirical moments. The screening cost parameter (c) is calibrated to match an aggregate unemployment rate of 15.1%. The scale and composition of the informal sector are governed by two costs: the

[19] Bonfiglioli and Gancia, 2019 report a range of values between 0.023 and 0.113 for $\gamma = 0.5$.

fixed operational informal cost (c_i), disciplined by the informal worker share (35.4%), and the convex informality cost (τ_i), targeted to match the informal firm share (69.8%). Selection into international markets is disciplined by the fixed exporting cost (c_x), matching an exporter share of 5.9%. To provide additional discipline on the model's selection mechanisms and the size distribution of top firms, we include the export sales share (15.5%) as a fifth, overidentifying moment.

Table II: Internally Calibrated Parameters

Parameter		**Value**
Free entry cost	f_e	0.707
Search cost level	α_0	5
Search cost curvature	α_1	7
Screening cost curvature	δ	1.848
Education cost	ε	0.46
Effort cost	η	0.088
Screening cost level	c	0.532
Informal-sector fixed cost	c_i	0.485
Informal-sector variable cost	τ_i	0.106
Exporting fixed cost	c_x	0.585

3.2 Model fit: moments generated by the model vs data

This section compares the main moments generated by the model with those computed using data from 2003 (see Dix-Carneiro et al., 2026; Ulyssea, 2018), which is the last year available for ECINF. Table III shows that our model matches well both labor market outcomes and measures of trade exposure, which are central to the core mechanism. However, the model underestimates the dispersion of firm size in the formal sector.[20] There is a tension between matching firm size distribution and the reported moments. A possible solution would follow Ulyssea, 2018 and introduce uncertainty about effective productivity at the time of the formalization decision: if firms observe only a noisy signal of θ before choosing their sector, some high-productivity firms receive a low signal and remain informal, while some low-productivity firms receive a high signal and formalize. This generates an overlap in the productivity distributions of formal and informal firms, increasing the average size of informal firms and reducing it for formal firms, thereby producing greater within-sector

[20]For instance, it implies a coefficient of variation of formal firm employment of 1.25, below the data value of 1.81 (trimming the top and bottom 1%).

dispersion. However, this extension would require two additional parameters and, critically, breaks the closed-form expressions for the screening intensity that underpin the theoretical results. We therefore retain the full-information assumption as the tractability cost of preserving the analytical structure of the model.

Table III: Data and Model Moments

Moments	**Data**	**Model**	**Source**
Share of informal firms (among all)	0.698	0.664	RAIS + ECINF
Share of informal workers	0.354	0.346	PNAD
Share of unemployment	0.151	0.153	PME
Share of exporters (among formal manufacturing)	0.059	0.056	RAIS + SECEX
Share of export sales (total formal manufacturing)	0.155	0.168	SECEX + IBGE

Notes: Data sources: RAIS (*Relação Anual de Informações Sociais*) is an administrative dataset collected annually by the Ministry of Labor, covering formal firms and workers. ECINF (*Pesquisa de Economia Informal Urbana*). It is conducted by IBGE (*Instituto Brasileiro de Geografia e Estatística*), Brazil's National Statistics Agency. It is a repeated cross-section survey focusing on small firms with up to five employees. The PNAD is the *Pesquisa Nacional por Amostra de Domicílios* (National Household Survey), a large household survey representative at the national level, also conducted by the IBGE. PME (*Pesquisa Mensal de Emprego*) is a monthly employment survey providing labor market indicators for major metropolitan areas. SECEX (*Secretaria de Comércio Exterior*) provides data on Brazilian exports and imports.

3.3 Trade liberalization: simulation results

We analyze the behavior of the model in response to a symmetric reduction in the iceberg trade cost τ_x from the baseline value 2.4 to 2.0, following Dix-Carneiro et al., 2026 and corresponding to a 16.7% reduction in trade costs. This shock increases the share of export sales to 19.2%, in line with recent Brazilian data (World Bank). The reduction in τ_x should be interpreted as a thought experiment: it asks what would happen to an economy with Brazil's structural characteristics if it were exposed to a reduction in export trade costs of this magnitude. As in Section 2.10, we assume that the economy initially operates in the low-effort equilibrium.

Table IV shows the main outcomes for each value of the trade costs. In particular, the first two columns report the outcomes for the baseline value $\eta = \eta_{\max} = 0.088$, that is, the upper bound consistent with equilibrium multiplicity, while the last two columns report the outcomes for $\eta = \eta_{\min} = 0.044$, that is, the lower bound.

We can connect the quantitative results discussed here to figure V. We first consider $\eta = \eta_{\max}$. This is point 2 in the figure. In this case, the trade shock is insufficient to trigger an equilibrium switch. The economy remains in the low-effort equilibrium, consistent with being far from the tipping region identified in Figure V. A substantially larger shock would

Table IV: Trade liberalization: within-equilibrium effects vs. equilibrium switch

	$\eta = \eta_{\max}$		$\eta = \eta_{\min}$	
	$\tau_x = 2.4$	$\tau_x = 2.0$	$\tau_x = 2.4$	$\tau_x = 2.0$
Effort	Low	Low	Low	High
Share of exporters	0.056	0.067	0.056	0.137
Share of export sales	0.168	0.192	0.168	0.173
Average screening cutoff	1.002	1.006	1.002	1.065
Average ability (formal)	2.475	2.486	2.475	2.841
Average ability (aggregate)	2.471	2.472	2.471	2.655
Aggregate unemployment	0.153	0.156	0.153	0.169
Entry cutoff	1.769	1.776	1.769	2.100
Formal/informal cutoff	2.439	2.439	2.439	2.156
Share of informal firms	0.664	0.659	0.664	0.086
Share of informal workers	0.346	0.337	0.346	0.021
Welfare	1.765	1.804	1.765	4.118

be required to move to the high-effort equilibrium. Trade liberalization increases export participation and export sales, which raises screening slightly and leads to a modest increase in average ability. This operates through a standard selection channel: low-productivity firms and low-ability matches exit. Higher screening, in turn, generates higher unemployment, while higher equilibrium wages raise the entry cutoff (the domestic selection effect). The shares of both informal firms and informal workers decline only marginally, in line with the empirical evidence that large trade liberalization episodes in the 1980s and 1990s were not accompanied by a substantial contraction of the informal sector (see Dix-Carneiro et al., 2026). Welfare–as measured by ex-ante expected income–increases, but the overall effects remain quantitatively modest.

We now turn to $\eta = \eta_{\min}$. This is point 1 in figure V. Here, the same trade shock crosses a tipping point and eliminates the low-effort equilibrium. As a result, the economy transitions to the high-effort equilibrium. The effects are nonlinear and large. Average ability and the entry cutoff rise, and the formal/informal cutoff declines. Informality drops dramatically, effectively moving the economy out of the developing-country range. Due to tighter screening, the unemployment rate increases more. Welfare rises substantially (+133%).

These two polar cases speak directly to the controversy surrounding trade liberalization in developing economies. The $\eta = \eta_{\max}$ case–modest formality gains, tighter screening, limited welfare improvement–is the cautionary tale that critics of rapid trade liberalization point to. The $\eta = \eta_{\min}$ case is the development success story. Both arise from the same trade shock; what determines which one materializes is whether the economy is sufficiently close to the coordination threshold.

Let us take stock of the main results obtained from the quantitative analysis. The presence of complementarities generates a threshold effect. Trade has limited impact when the economy is far from the threshold but can induce large changes in exporting, human capital, and informality when the threshold is crossed. This pattern is consistent with a Big Push: a discrete, nonlinear transition rather than a gradual adjustment.

The mechanism underlying this result can be summarized as follows: (i) trade liberalization increases firm revenues; (ii) higher revenues raise optimal screening intensity; (iii) stronger screening increases the returns to human capital; (iv) workers respond by increasing effort; (v) if the economy is sufficiently close to the threshold, this feedback loop triggers a discrete shift to the high-effort equilibrium. Graphically, the trade shock shifts the effort best-response upward, potentially eliminating the low-effort equilibrium and leaving the high-effort equilibrium as the unique outcome.[21] The large effects in the $\eta_{\min}$ case reflect the economy being close to this tipping point. Introducing additional frictions that are absent in the current static setup would likely smooth the transition and attenuate the magnitude of the response, without altering the underlying mechanism.

4 Conclusion

We have shown that, under certain conditions, trade is not only reallocative–the main lesson from Melitz, 2003–but also equilibrium-switching: it can trigger a Big Push by shifting the economy from a low-human-capital, high-informality equilibrium to a high-human-capital, formal equilibrium. The central insight of our analysis is a novel mechanism linking trade, informality, and human capital: effort serves as a key margin of adjustment to trade shocks.

Calibrating the model to a representative developing economy, we find that the Big Push mechanism generates quantitatively large and nonlinear effects: trade has modest impact when the economy is far from the threshold, but produces large changes in human capital, formalization, and welfare when the economy is sufficiently close to it. Direct empirical testing of such equilibrium-switching effects lies beyond the scope of this paper. The mechanism generates testable predictions–in particular, the response of education and informality to trade exposure may be nonlinear–which we leave for future empirical work, exploiting the rich Brazilian matched employer–employee data.

Several extensions could potentially enrich the framework, including allowing for screening in the informal sector, modeling import-side trade liberalization, introducing capital

[21] We relegate the graphical analysis to Appendix K.

constraints, and incorporating heterogeneity in effort costs across workers.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process.

During the preparation of this work, the authors used ChatGPT and Claude to assist with proofreading, as well as code development, debugging, and optimization. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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Online Appendix

A Preferences and indirect utility

We assume that preferences are defined over an aggregate consumption index $\mathcal{C}$ and workers are risk-neutral:

$$U = \mathcal{C} \tag{A.1}$$

where $\mathcal{C} = Q$ and Q takes the same form as in 2.1.

Ex-ante expected indirect utility is therefore:

$$V = \mathbb{E}\left(\frac{w}{\mathcal{P}}\right) = \omega_i \tag{A.2}$$

where $\mathbb{E}$ is the expectations operator and $\mathcal{P}$ is the price index of the aggregate consumption index $\mathcal{C}$, which we choose as the numeraire ($\mathcal{P} = 1$). This is intuitive: ex-ante expected indirect utility is equal to the ex-ante expected wage, that is, the wage workers anticipate when they are homogeneous, prior to education, effort choices, and employment outcomes.

Ex-post indirect utility depends on *ex-post* wages, which depend on the worker's sector and firm of employment:

$$V = \begin{cases} \frac{w_f(\theta)}{\mathcal{P}} = w_f(\theta) & \text{if employed by a } \theta\text{-firm in the formal sector} \\ \frac{w_i(\theta)}{\mathcal{P}} = w_i & \text{if employed by any firm in the informal sector} \\ 0 & \text{if unemployed.} \end{cases}$$

B Derivation of profits and revenue

<u>Formal sector.</u> Profit as a function of revenue is expressed as

$$\pi_f(\theta) = \frac{\Gamma(1-\tau_f)}{1+\beta\gamma} r_f(\theta) - c_f \tag{A.3}$$

where $\Gamma \equiv 1 - \beta\gamma - \frac{\beta(1-\gamma k)}{\delta} > 0$.

We can solve explicitly for

$$\frac{(1-\tau_f)\beta\gamma}{1+\beta\gamma} r_f(\theta) = \phi_1 \phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}} \tag{A.4}$$

$$n_f(\theta) = \phi_1 \phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma+\Gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}} \tag{A.5}$$

$$a_c(\theta) = \phi_1^{\frac{1}{\delta}} \phi_2^{1-\beta\gamma} b_f^{-\frac{\beta\gamma}{\delta\Gamma}} c^{-\frac{1-\beta\gamma}{\delta\Gamma}} Q^{\frac{1-\beta}{\delta\Gamma}} \theta^{\frac{\beta}{\delta\Gamma}} \tag{A.6}$$

$$h_f(\theta) = \phi_1^{1-\frac{k}{\delta}} \phi_2^{\beta-k} b_f^{\frac{\beta-\delta}{\delta\Gamma}} c^{\frac{k-\beta}{\delta\Gamma}} Q^{\frac{(1-\beta)(\delta-k)}{\delta\Gamma}} \theta^{\frac{\beta(\delta-k)}{\delta\Gamma}} \tag{A.7}$$

$$w_f(\theta) = \phi_1^{\frac{k}{\delta}} \phi_2^{(1-\beta\gamma)k} b_f^{\frac{1-\beta\gamma-\beta/\delta}{\Gamma}} c^{-\frac{(1-\beta\gamma)k}{\delta\Gamma}} Q^{\frac{(1-\beta)k}{\delta\Gamma}} \theta^{\frac{\beta k}{\delta\Gamma}} \tag{A.8}$$

where we have introduced the constants $\phi_1 \equiv \left[\frac{(1-\tau_f)\beta\gamma}{1+\beta\gamma}\left(\frac{k}{k-1}\right)^{\beta}\right]^{\frac{1}{\Gamma}}$ and $\phi_2 \equiv \left(\frac{1-\gamma k}{\gamma}\right)^{\frac{1}{\delta\Gamma}}$.

The relative revenue of any two firms is $\frac{r_f(\theta')}{r_f(\theta'')} = \left(\frac{\theta'}{\theta''}\right)^{\frac{\beta}{\Gamma}}$.

The profit function can then be expressed as

$$\pi_f(\theta) = \frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}} - c_f. \tag{A.9}$$

Informal sector. Profit as a function of revenue is expressed as

$$\pi_i(\theta) = \frac{(1-\beta\gamma)\theta^{-\tau_i\beta}}{1+\beta\gamma} r_i(\theta) - c_i. \tag{A.10}$$

We can solve explicitly for

$$r_i(\theta) = \phi_3^{\beta\gamma} b_i^{-\frac{\beta\gamma}{1-\beta\gamma}} Q^{\frac{1-\beta}{1-\beta\gamma}} \theta^{\frac{\beta(1-\beta\gamma\tau_i)}{1-\beta\gamma}} \tag{A.11}$$

$$n_i(\theta) = \phi_3 b_i^{-\frac{1}{1-\beta\gamma}} Q^{\frac{1-\beta}{1-\beta\gamma}} \theta^{\frac{\beta(1-\tau_i)}{1-\beta\gamma}} \tag{A.12}$$

where we have introduced the constant $\phi_3 \equiv \left(\frac{\beta\gamma}{1+\beta\gamma}\right)^{\frac{1}{1-\beta\gamma}}$.

The relative revenue of any two firms is therefore equal to $\frac{r_i(\theta')}{r_i(\theta'')} = \left(\frac{\theta'}{\theta''}\right)^{\frac{\beta(1-\beta\gamma\tau_i)}{1-\beta\gamma}}$. After solving for revenue, profit can be rewritten as a function of productivity:

$$\pi_i(\theta) = \frac{1-\beta\gamma}{1+\beta\gamma}\phi_3^{\beta\gamma} b_i^{-\frac{\beta\gamma}{1-\beta\gamma}} Q^{\frac{1-\beta}{1-\beta\gamma}} \theta^{\frac{\beta(1-\tau_i)}{1-\beta\gamma}} - c_i. \tag{A.13}$$

From (A.13), the zero-profit condition (ZPC) is

$$\frac{1-\beta\gamma}{1+\beta\gamma}\phi_3^{\beta\gamma} b_i^{-\frac{\beta\gamma}{1-\beta\gamma}} Q^{\frac{1-\beta}{1-\beta\gamma}} (\theta_i^*)^{\frac{\beta(1-\tau_i)}{1-\beta\gamma}} = c_i. \tag{A.14}$$

C Formalization decision in the closed economy

Potential entrants pay the entry cost $f_e > 0$ (in units of the numeraire) and randomly draw θ from a Pareto distribution. The value of a firm is given by

$$\begin{aligned} v(\theta) &= \max\left\{0, \sum_{t=0}^{\infty}(1-d)^t \max\{\pi_i(\theta), \pi_f(\theta)\}\right\} \\ &= \max\left\{0, \frac{1}{d}\max\{\pi_i(\theta), \pi_f(\theta)\}\right\} \end{aligned}$$

where d is the exogenous death rate.

Firms with $\theta < \theta_i^*$, such that $\pi(\theta_i^*) < 0$, exit; firms with $\theta \geq \theta_f^*$, such that $\pi_f(\theta_f^*) = \pi_i(\theta_f^*)$, become formal; firms with $\theta \in [\theta_i^*, \theta_f^*]$ become informal. Using equation A.14, we can pin down the cutoff for the informal sector: $\theta_i^* = \left(\frac{c_i}{\frac{1-\beta\gamma}{1+\beta\gamma}\phi_3^{\beta\gamma} b_i^{-\frac{\beta\gamma}{1-\beta\gamma}} Q^{\frac{1-\beta}{1-\beta\gamma}}}\right)^{\frac{1-\beta\gamma}{\beta(1-\tau_i)}}$. We cannot solve analytically for θ_f^* such that

$$\frac{1-\beta\gamma}{1+\beta\gamma}\phi_3^{\beta\gamma} b_i^{-\frac{\beta\gamma}{1-\beta\gamma}} Q^{\frac{1-\beta}{1-\beta\gamma}} \theta_f^{*\frac{\beta(1-\tau_i)}{1-\beta\gamma}} - c_i = \frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} \theta_f^{*\frac{\beta}{\Gamma}} - c_f,$$

unless we make some simplifying assumptions, so we must resort to numerical methods. More specifically, given $\xi_i \equiv \frac{1-\beta\gamma}{1+\beta\gamma}\phi_3^{\beta\gamma} b_i^{-\frac{\beta\gamma}{1-\beta\gamma}} Q^{\frac{1-\beta}{1-\beta\gamma}}$, $\xi_f \equiv \frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}}$, $s \equiv \frac{\beta(1-\tau_i)}{1-\beta\gamma}$, $t \equiv \frac{\beta}{\Gamma}$, and $\Delta c \equiv c_i - c_f$, the equation above becomes

$$\xi_i \theta_f^{*s} - \xi_f \theta_f^{*t} = \Delta c, \tag{A.15}$$

which is transcendental in θ_f^*. If the exponents are equal, $s = t$, then A.15 reduces to

$$(\xi_i - \xi_f)\,\theta_f^{*s} = \Delta c.$$

Provided that $\xi_i - \xi_f \neq 0$ and the right-hand side has the correct sign for a real solution, the solution is

$$\theta_f^* = \left(\frac{\Delta c}{\xi_i - \xi_f}\right)^{1/s}.$$

If instead $\Delta c = 0$, A.15 becomes

$$\xi_i \theta_f^{*s} = \xi_f \theta_f^{*t}.$$

This implies

$$\theta_f^{*s-t} = \frac{\xi_f}{\xi_i},$$

and therefore, if $\frac{\xi_f}{\xi_i} > 0$, the solution is

$$\theta_f^* = \left(\frac{\xi_f}{\xi_i}\right)^{1/(s-t)}.$$

In the generic case, $s \neq t$ and $\Delta c \neq 0$, equation A.15 admits no closed-form solution in general, except for some exponent relations.

The free entry condition (FEC), which equates the expected value of entry to the sunk entry cost, can be written as

$$\int_{\theta_i^*}^{\theta_f^*} \frac{\pi_i(\theta)}{d} dG(\theta) + \int_{\theta_f^*}^{\infty} \frac{\pi_f(\theta)}{d} dG(\theta) = f_e.$$

To ensure integrability, using A.9 and A.13, it must be that $z > \max\left\{\frac{\beta(1-\tau_i)}{1-\beta\gamma}, \frac{\beta}{\Gamma}\right\}$ and $z \geq 1$.

D Proof of Proposition 1

First, we consider the behavior of firms. Firms know k and choose screening intensity according to (A.6). The condition for $a_c(\theta, k_1) > a_c(\theta, k_0)$ is obtained by comparing a_c across equilibria using (A.6):

$$\frac{a_c(\theta,k_1)}{a_c(\theta,k_0)} = \frac{\phi_1(k_1)^{1/\delta}\,\phi_2(k_1)^{(1-\beta\gamma)/(\delta\Gamma_1)}}{\phi_1(k_0)^{1/\delta}\,\phi_2(k_0)^{(1-\beta\gamma)/(\delta\Gamma_0)}} \cdot \frac{b_{f,0}^{\beta\gamma/(\delta\Gamma_0)}}{b_{f,1}^{\beta\gamma/(\delta\Gamma_1)}} \cdot c^{-\frac{1-\beta\gamma}{\delta}\left(\frac{1}{\Gamma_1}-\frac{1}{\Gamma_0}\right)} \cdot \frac{Q_1^{(1-\beta)/(\delta\Gamma_1)}}{Q_0^{(1-\beta)/(\delta\Gamma_0)}} \cdot \theta^{\frac{\beta}{\delta}\left(\frac{1}{\Gamma_1}-\frac{1}{\Gamma_0}\right)}. \tag{A.16}$$

The terms ϕ_1 and ϕ_2 evaluated at k_1 both exceed their k_0 counterparts since $k_1 < k_0$ implies larger bases and larger exponents ($\Gamma_1 < \Gamma_0$). To prove that (A.16) is greater than one therefore boils down to

$$\frac{b_{f,0}^{\beta\gamma/(\delta\Gamma_0)}}{b_{f,1}^{\beta\gamma/(\delta\Gamma_1)}} \cdot c^{-\frac{1-\beta\gamma}{\delta}\left(\frac{1}{\Gamma_1}-\frac{1}{\Gamma_0}\right)} \cdot \frac{Q_1^{(1-\beta)/(\delta\Gamma_1)}}{Q_0^{(1-\beta)/(\delta\Gamma_0)}} > 1, \tag{A.17}$$

which can be equivalently written as (25).

Next, we consider the choice of workers and study under what conditions, given the optimal screening cutoff conditional on k, a deviation is not individually profitable.

The expected wage of high-effort conditional on being sampled under low-screening intensity is given by:

$$\begin{aligned}
\mathbb{E}[w_f(k_1, a_c(k_0))] &= \frac{\int_{\theta_0^*}^{\infty} w_f(\theta,k_0) a_c(\theta,k_0)^{-k_1} dG(\theta)}{1-G(\theta_0^*)} \frac{N_{f,0}}{L_{f,0}} \\
&\overset{22}{=} \frac{\int_{\theta_0^*}^{\infty} a_c(\theta,k_0)^{k_0-k_1} d(1-\theta^{-z})}{(\theta_0^*)^{-z}} b_f \frac{N_{f,0}}{L_{f,0}} \\
&\overset{23}{=} z\frac{\int_{\theta_0^*}^{\infty} \phi^{k_0-k_1}(\theta)^{\frac{\beta(k_0-k_1)}{\delta\Gamma_0}}(\theta)^{-z-1} d(\theta)}{(\theta_0^*)^{-z}} b_f \frac{N_{f,0}}{L_{f,0}} \\
&\overset{24}{=} z\frac{\phi^{k_0-k_1}(\theta_0^*)^{\frac{\beta(k_0-k_1)}{\delta\Gamma_0}-z}}{z-\frac{\beta(k_0-k_1)}{\delta\Gamma_0}} (\theta_0^*)^z b_f \frac{N_{f,0}}{L_{f,0}} \\
&= \frac{\delta z\Gamma_0 a_c(\theta_0^*,k_0)^{\Delta k}}{\delta z\Gamma_0 - \beta\Delta k} b_f \frac{N_{f,0}}{L_{f,0}}.
\end{aligned}$$

The expected wage of low-effort conditional on being sampled under high-screening in-

[22] using (4), $w_f(\theta,k) = b_f a_c(\theta,k)^k$

[23] using A.6, $a_c(\theta) = \phi\theta^{\frac{\beta}{\delta\Gamma}}$ where $\phi \equiv \phi_1^{\frac{1}{\delta}}\phi_2^{1-\beta\gamma} b_f^{-\frac{\beta\gamma}{\delta\Gamma}} c^{-\frac{1-\beta\gamma}{\delta\Gamma}} Q^{\frac{1-\beta}{\delta\Gamma}}$

[24] $\frac{\beta(k_0-k_1)}{\delta\Gamma_0} - z < 0$ as, by assumption, $z\Gamma_0 > 1 > \frac{\beta k_0}{\delta} > \frac{\beta(k_0-k_1)}{\delta}$

tensity is given by:

$$
\begin{aligned}
\mathbb{E}[w_f(k_0, a_c(k_1))] &= \frac{\int_{\theta_1^*}^{\infty} w_f(\theta, k_1) a_c(\theta, k_1)^{-k_0} dG(\theta)}{1 - G(\theta_1^*)} \frac{N_{f,1}}{L_{f,1}} \\
&= \frac{\int_{\theta_1^*}^{\infty} a_c(\theta, k_1)^{k_1 - k_0} d(1 - \theta^{-z})}{(\theta_1^*)^{-z}} b_f \frac{N_{f,1}}{L_{f,1}} \\
&= z \frac{\int_{\theta_1^*}^{\infty} \phi^{k_1 - k_0} (\theta)^{\frac{\beta(k_1 - k_0)}{\delta\Gamma_1}} (\theta)^{-z-1} d(\theta)}{(\theta_1^*)^{-z}} b_f \frac{N_{f,1}}{L_{f,1}} \\
&= z \frac{\phi^{k_1 - k_0} (\theta_1^*)^{\frac{\beta(k_1 - k_0)}{\delta\Gamma_1} - z}}{z - \frac{\beta(k_1 - k_0)}{\delta\Gamma_1}} (\theta_1^*)^z b_f \frac{N_{f,1}}{L_{f,1}} \\
&= \frac{\delta z \Gamma_1 a_c(\theta_1^*, k_1)^{-\Delta k}}{\delta z \Gamma_1 + \beta \Delta k} b_f \frac{N_{f,1}}{L_{f,1}}.
\end{aligned}
$$

Therefore, deviating from the low-effort equilibrium is not profitable if

$$
b_f \frac{N_{f,0}}{L_{f,0}} = \mathbb{E}[w_f(k_0, a_c(k_0))] > \mathbb{E}[w_f(k_1, a_c(k_0))] - \eta. \tag{A.18}
$$

Similarly, deviating from the high-effort equilibrium is not profitable if

$$
b_f \frac{N_{f,1}}{L_{f,1}} - \eta = \mathbb{E}[w_f(k_1, a_c(k_1))] - \eta > \mathbb{E}[w_f(k_0, a_c(k_1))]. \tag{A.19}
$$

Combining (A.18) and (A.19) yields (26). Multiplicity requires a non-empty set $[A_{min}, A_{max}]$:

$$
\frac{A_{max}}{A_{min}} = \left(\frac{\delta z \Gamma_1 + \beta \Delta k}{\delta z \Gamma_1}\right) \left(\frac{\delta z \Gamma_0 - \beta \Delta k}{\delta z \Gamma_0}\right) \left(\frac{\Gamma_0(1 - \gamma k_1)}{\Gamma_1(1 - \gamma k_0)}\right)^{\frac{\Delta k}{\delta}} > 1.
$$

E Proof of Corollary 1

The good equilibrium is sustainable if and only if

$$
\mathbb{E}[w_f(k_1, a_c(k_1))] - \eta > \mathbb{E}[w_f(k_0, a_c(k_1))],
$$

that is, if the expected wage from high effort net of the effort cost (term on the left hand side) exceeds the value of deviating and exerting low effort (term on the right hand side). This inequality can be equivalently written as

$$
R(a_c(k_1)) \equiv \mathbb{E}[w_f(k_1) - w_f(k_0) | a_c(k_1)] > \eta. \tag{A.20}
$$

In a similar way, the bad equilibrium is unsustainable if and only if

$$
\mathbb{E}[w_f(k_1, a_c(k_0))] - \eta > \mathbb{E}[w_f(k_0, a_c(k_0))],
$$

which can be equivalently written as condition (27).

Necessity. Suppose $R(a_c(k_0)) \leq \eta$. A worker in the bad equilibrium expects firms to screen at intensity $a_c(\theta, k_0)$. The expected wage premium from deviating to high effort is $R(a_c(k_0)) \leq \eta$, so deviation is not profitable. The bad equilibrium is therefore sustainable and no Big Push occurs.

Sufficiency. Suppose $R(a_c(k_0)) > \eta$. A worker in the bad equilibrium who expects firms to screen at $a_c(\theta, k_0)$ now finds deviation to high effort strictly profitable, so the bad equilibrium is unsustainable. It remains to show that the good equilibrium is itself a valid fixed point, i.e. that condition (A.20) holds. By condition (25), $a_c(\theta, k_1) > a_c(\theta, k_0)$ for all $\theta \geq \theta_f^*$, and $R(\cdot)$ is strictly increasing in a_c, so

$$R(a_c(k_1)) > R(a_c(k_0)) > \eta.$$

Hence a worker in the good equilibrium, expecting screening at intensity $a_c(\theta, k_1)$, finds deviation to low effort unprofitable. The good equilibrium is self-sustaining and, since the bad equilibrium is simultaneously unsustainable, it is the unique outcome.

F Derivation of the equilibrium condition

Let $T^a = M\left(\int_{\theta_f^*}^{\infty} \tau_f r_f(\theta) dG(\theta) + \int_{\theta_i^*}^{\theta_f^*} r_i(\theta)(1-\theta^{-\tau_i\beta}) dG(\theta)\right)$ denote aggregate tax revenue in autarky and $R^a = M\left(\frac{1-\tau_f}{1+\beta\gamma}\int_{\theta_f^*}^{\infty} r_f(\theta) dG(\theta) + \frac{1}{1+\beta\gamma}\int_{\theta_i^*}^{\theta_f^*} \theta^{-\tau_i\beta} r_i(\theta) dG(\theta)\right)$ aggregate firm revenue net of taxes. It holds that

$$\begin{aligned}
E &= L_i\omega_i + L_f(\omega_i + \varepsilon + \mathbb{I}_\eta \eta) + T^a + R^a \\
E &= \omega_i(L_i + L_f) + L_f(\varepsilon + \mathbb{I}_\eta\eta) + \frac{\beta\gamma}{1+\beta\gamma} M\left(\int_{\theta_f^*}^{\infty} \tau_f r_f(\theta)\, dG(\theta) + \int_{\theta_i^*}^{\theta_f^*} r_i(\theta)(1-\theta^{-\tau_i\beta})\, dG(\theta)\right) \\
&\qquad + \frac{M}{1+\beta\gamma}\left(\int_{\theta_f^*}^{\infty} r_f(\theta)\, dG(\theta) + \int_{\theta_i^*}^{\theta_f^*} r_i(\theta)\, dG(\theta)\right) \\
E &= \omega_i L + L_f(\varepsilon + \mathbb{I}_\eta\eta) + \frac{\beta\gamma}{1+\beta\gamma} T^a + \frac{1}{1+\beta\gamma} E
\end{aligned}$$

which yields

$$\frac{\beta\gamma}{1+\beta\gamma} Q - L_f(\varepsilon + \mathbb{I}_\eta\eta) = \omega_i \bar{L} + \frac{\beta\gamma}{1+\beta\gamma} T^a \tag{A.21}$$

where the last equality follows from $E = Q$ and the labor market clearing condition $L = \bar{L}$.

G Open economy equilibrium

In the open economy, revenue of formal firms is given by

$$r_f(\theta) = Y(\theta)^{1-\beta} A y_f(\theta)^{\beta}.$$

We can solve for

$$\frac{(1-\tau_f)\beta\gamma}{1+\beta\gamma} r_f(\theta) = \phi_1 \phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} Y(\theta)^{\frac{1-\beta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}} \tag{A.22}$$

$$n_f(\theta) = \phi_1 \phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma+\Gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} Y(\theta)^{\frac{1-\beta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}} \tag{A.23}$$

$$a_c(\theta) = \phi_1^{\frac{1}{\delta}} \phi_2^{1-\beta\gamma} b_f^{-\frac{\beta\gamma}{\delta\Gamma}} c^{-\frac{1-\beta\gamma}{\delta\Gamma}} Q^{\frac{1-\beta}{\delta\Gamma}} Y(\theta)^{\frac{1-\beta}{\delta\Gamma}} \theta^{\frac{\beta}{\delta\Gamma}} \tag{A.24}$$

$$h_f(\theta) = \phi_1^{1-\frac{k}{\delta}} \phi_2^{\beta-k} b_f^{\frac{\beta-\delta}{\delta\Gamma}} c^{\frac{k-\beta}{\delta\Gamma}} Q^{\frac{(1-\beta)(\delta-k)}{\delta\Gamma}} Y(\theta)^{\frac{(1-\beta)(\delta-k)}{\delta\Gamma}} \theta^{\frac{\beta(\delta-k)}{\delta\Gamma}} \tag{A.25}$$

$$w_f(\theta) = \phi_1^{\frac{k}{\delta}} \phi_2^{(1-\beta\gamma)k} b_f^{\frac{1-\beta\gamma-\beta/\delta}{\Gamma}} c^{-\frac{(1-\beta\gamma)k}{\delta\Gamma}} Q^{\frac{(1-\beta)k}{\delta\Gamma}} Y(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} \theta^{\frac{\beta k}{\delta\Gamma}}. \tag{A.26}$$

The profit function of exporters is given by

$$\pi_x(\theta) = \frac{\Gamma}{\beta\gamma} \phi_1 \phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} Y_x^{\frac{1-\beta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}} - c_x - c_f. \tag{A.27}$$

H Formalization decision in the open economy

As in Section C, potential entrants pay the entry cost $f_e > 0$ (in units of the numeraire) and randomly draw θ from a Pareto distribution. The value of a firm is given by

$$\begin{aligned} v(\theta) &= \max\left\{0, \sum_{t=0}^{\infty}(1-d)^t \max\left\{\pi_i(\theta), \max\left\{\pi_f(\theta), \pi_x(\theta)\right\}\right\}\right\} \\ &= \max\left\{0, \frac{1}{d} \max\left\{\pi_i(\theta), \max\left\{\pi_f(\theta), \pi_x(\theta)\right\}\right\}\right\} \end{aligned}$$

where d is the exogenous death rate.

Firms with $\theta < \theta_i^*$, such that $\pi(\theta_i^*) < 0$, exit; firms with $\theta \in [\theta_i^*, \theta_f^*]$ become informal; firms with $\theta \in [\theta_f^*, \theta_x^*]$, such that $\pi_f(\theta_x^*) = \pi_x(\theta_x^*)$, become formal domestic producers; firms with $\theta \geq \theta_x^*$ become exporters.

The cutoff for the informal sector is given, as in Section C, by $\theta_i^* = \left(\frac{c_i}{\frac{1-\beta\gamma}{1+\beta\gamma}\phi_3^{\beta\gamma} b_i^{-\frac{\beta\gamma}{1-\beta\gamma}} Q^{\frac{1-\beta}{1-\beta\gamma}}}\right)^{\frac{1-\beta\gamma}{\beta(1-\tau_i)}}$. The cutoff θ_f^* does not admit a closed-form solution, but we can pin down the cutoff for exporting: $\theta_x^* = \left(\frac{c_x}{\frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)} b_f^{-\frac{\beta\gamma}{\Gamma}} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} Q^{\frac{1-\beta}{\Gamma}} \left(Y_x^{\frac{1-\beta}{\Gamma}} - 1\right)}\right)^{\frac{\Gamma}{\beta}}$.

The FEC can be written as:

$$\int_{\theta_i^*}^{\theta_f^*} \frac{\pi_i(\theta)}{d} dG(\theta) + \int_{\theta_f^*}^{\theta_x^*} \frac{\pi_f(\theta)}{d} dG(\theta) + \int_{\theta_x^*}^{\infty} \frac{\pi_x(\theta)}{d} dG(\theta) = f_e. \tag{A.28}$$

To ensure integrability, using equation A.9 and A.13, it must be that $z > \max\left\{\frac{\beta(1-\tau_i)}{1-\beta\gamma}, \frac{\beta}{\Gamma}\right\}$ and $z \geq 1$.

I Equilibrium condition in the open economy

The mass of firms is determined by

$$E = M\left(\int_{\theta_i^*}^{\theta_f^*} r_i(\theta)dG(\theta) + \int_{\theta_f^*}^{\infty} r_d(\theta)dG(\theta)\right) + \tilde{M}\int_{\tilde{\theta}_x^*}^{\infty} \tilde{r}_x(\theta)dG(\theta). \tag{A.29}$$

Let $T = M\left(\tau_f\left(\int_{\theta_f^*}^{\theta_x^*} r_d(\theta)dG(\theta) + \int_{\theta_x^*}^{\infty} r_x(\theta)dG(\theta)\right) + \int_{\theta_i^*}^{\theta_f^*} r_i(\theta)(1-\theta^{-\tau_i\beta})dG(\theta)\right)$ denote aggregate tax revenue and $R = M\left(\frac{1-\tau_f}{1+\beta\gamma}\left(\int_{\theta_f^*}^{\theta_x^*} r_d(\theta)dG(\theta) + \int_{\theta_x^*}^{\infty} r_x(\theta)dG(\theta)\right) + \frac{1}{1+\beta\gamma}\int_{\theta_i^*}^{\theta_f^*} \theta^{-\tau_i\beta} r_i(\theta)dG(\theta)\right)$ aggregate firm revenue after taxes.

Using country symmetry and labor market clearing ($L = \bar{L}$), we obtain an equilibrium condition analogous to that in the closed-economy setting:

$$\begin{aligned}
E &= L_i\omega_i + L_f(\omega_i + \varepsilon + \mathbb{I}_\eta\eta) + T + R \\
E &= \omega_i(L_i + L_f) + L_f(\varepsilon + \mathbb{I}_\eta\eta) + \frac{\beta\gamma}{1+\beta\gamma}M\Bigg[\tau_f\left(\int_{\theta_f^*}^{\theta_x^*} r_d(\theta)\,dG(\theta) + \int_{\theta_x^*}^{\infty} r_x(\theta)\,dG(\theta)\right) \\
&\quad + \int_{\theta_i^*}^{\theta_f^*} r_i(\theta)\left(1-\theta^{-\tau_i\beta}\right)\,dG(\theta)\Bigg] \\
&\quad + \frac{M}{1+\beta\gamma}\left(\int_{\theta_i^*}^{\theta_f^*} r_i(\theta)\,dG(\theta) + \int_{\theta_f^*}^{\infty} r_d(\theta)\,dG(\theta) + \int_{\theta_x^*}^{\infty} r_x(\theta)\,dG(\theta)\right) \\
E &= \omega_i L + L_f(\varepsilon + \mathbb{I}_\eta\eta) + \frac{\beta\gamma}{1+\beta\gamma}T + \frac{1}{1+\beta\gamma}E
\end{aligned}$$

which yields

$$\frac{\beta\gamma}{1+\beta\gamma}Q - L_f(\varepsilon + \mathbb{I}_\eta\eta) = \omega_i\bar{L} + \frac{\beta\gamma}{1+\beta\gamma}T. \tag{A.30}$$

J Proof of Proposition 2

We proceed in three steps.

Step 1 (Export selection raises aggregate screening intensity). Under autarky, $\tau_x = \infty$ implies $\theta_x^* = \infty$, so $Y(\theta) = 1$ for all θ and no firm exports. As τ_x falls from ∞, the export cutoff θ_x^* becomes finite and a positive measure of formal firms with $\theta \geq \theta_x^*$ begins to export, experiencing a discrete jump in revenue: by (A.22), $r_f(\theta)$ increases proportionally to $Y_x^{(1-\beta)/\Gamma} > 1$ for every exporter. By (A.24), the optimal screening cutoff satisfies $a_c(\theta) \propto r_f(\theta)^{1/\delta}$, so $a_c(\theta)$ increases for all exporters, which raises $R(a_c(k_0))$ as τ_x falls below ∞.

Step 2 (Existence of a threshold trade cost). Under autarky the economy is in the bad equilibrium by assumption, so $R(a_c(k_0)) \leq \eta$ at $\tau_x = \infty$. By Step 1, $R(a_c(k_0))$ is strictly increasing as τ_x decreases. By continuity of $R(a_c(k_0))$ in τ_x, there exists a threshold $\bar{\tau}_x \in (1, \infty)$ such that

$$R(a_c(k_0)) \begin{cases} \leq \eta & \text{if } \tau_x \geq \bar{\tau}_x, \\ > \eta & \text{if } \tau_x < \bar{\tau}_x. \end{cases}$$

For any $\tau_x < \bar{\tau}_x$, condition (27) holds and Corollary 1 applies: the bad equilibrium is unsustainable and workers switch to high effort, shifting the ability distribution from k_0 to $k_1 < k_0$.

Step 3 (Self-consistency of the good equilibrium). We show that, when workers exert high effort ($k = k_1$), the screening decisions of *all* formal firms–not only exporters–are mutually consistent with $k = k_1$, so that the good equilibrium is a stable fixed point in the open economy. Given $k = k_1$, each formal firm with productivity θ optimally sets its screening cutoff according to (A.24), yielding $a_c(\theta, k_1)$. Condition (25) ensures that $a_c(\theta, k_1) > a_c(\theta, k_0)$ for every formal firm, including purely domestic ones whose revenues decline due to higher equilibrium wages: the fall in k from k_0 to k_1 increases the dispersion of the ability distribution, raising the marginal return to screening for every formal firm by enough to dominate the negative revenue effect. Since all formal firms screen at $a_c(\theta, k_1)$, the expected return to high effort is $R(a_c(k_1))$. By the strict monotonicity of $R(\cdot)$,

$$R(a_c(k_1)) > R(a_c(k_0)) > \eta,$$

where the second inequality holds because $\tau_x < \bar{\tau}_x$. Therefore no worker finds it profitable to deviate to low effort. The pair $\{k = k_1,\ a_c(\theta, k_1) \text{ for all } \theta \geq \theta_f^*\}$ is therefore a stable fixed point: workers' high-effort choice is optimal given firms' screening decisions, and firms' screening decisions are optimal given the ability distribution induced by high effort. Since condition (27) simultaneously makes the bad equilibrium unsustainable, the good equilibrium is the unique outcome.

K Continuous effort choice

For the sake of tractability, we have modeled effort as a binary choice. How would our results change if workers could choose effort e from a continuum? The outcome would depend on assumptions about the form of the effort choice function. The role of the binary choice is to generate in a tractable way two discrete states for the economy: a bad state where everyone exerts low effort and a good state where everyone exerts high effort. The economy can get stuck in the bad equilibrium because no single agent finds it profitable to deviate. Trade acts as a shock that can switch the economy from the bad equilibrium to the good one: this generates a discrete jump. However, this is not the only way to generate multiple equilibria. To study the equilibrium determination and the possibility of multiplicity, consider the optimal effort choice (e^*) of an individual worker given firms' screening intensity. The incentive to exert effort is increasing in workers' heterogeneity and therefore in average effort ($\bar{e}$). Then, the worker's best-response function is given by $e^* = T(\bar{e})$, which maps the average effort ($\bar{e}$) into the optimal effort of an individual (e^*). An equilibrium is a fixed point of the mapping $\bar{e} = T(\bar{e})$. In figure A.1, we plot three response functions: in the first panel, a step function, representing our binary effort choice; in the second panel, a continuous S-shaped function; in the third panel, a continuous concave function. When we use the step function ($\bar{e} = \{0, 1\}$), we obtain two stable equilibria: $e^* = 0$ for $\bar{e} < e_{crit}$ and $e^* = 1$ for $\bar{e} > e_{crit}$. The interior tipping point e_{crit} is an unstable equilibrium where workers are indifferent between the two discrete effort choices. It is a critical threshold defining two sets of incentives. If the best response function is S-shaped, multiple equilibria are also possible: two stable equilibria (low effort and high effort) arise along with one unstable interior equilibrium in which workers are just indifferent. The best-response function crosses the 45-degree line ($e^* = \bar{e}$) three times: at low effort, where the curve is flat, $T'(\bar{e}) < 1$, as firms screen very little and the private return to high effort is low; at intermediate effort, where the curve becomes steeper than the 45-degree line, $T'(\bar{e}) > 1$, and the complementarity is strongest; at high effort, where the curve flattens again because of diminishing returns to screening. The tipping point is unstable because any perturbation, either positive or negative, will cause effort to shift away from this point toward either of the outer equilibria. The Big Push mechanism remains intact. A trade shock shifts this S-shaped curve upwards, potentially causing the low-effort equilibrium to vanish if the shock is large enough. This is precisely an equilibrium-switching phenomenon: a discontinuous jump from the bad to the good equilibrium. Instead, if the best response function were, for example, strictly concave, starting above the origin, it would cross the 45-degree line only once. This results in a unique stable equilibrium. The ability effect triggered by the trade shock would then be a marginal adjustment rather than an equilibrium-switching phenomenon. The Big Push mechanism would then disappear.

L Alternative distributional assumptions

Ability distribution. Following Helpman et al., 2010 and Bonfiglioli and Gancia, 2019, we assume a Pareto distribution for workers' ability. This choice is motivated not only by analytical convenience but also and importantly by the fat right tail: screening yields large gains from selecting top workers, and the lower shape parameter k implies more dispersion and higher conditional mean.

We consider now some alternative ability distributions ranked from weakest to strongest for the strength of effort–screening complementarities: normal, lognormal and Pareto-lognormal (PLN).

Suppose individual ability is normally distributed,

$$a \sim N(\mu, \sigma^2),$$

with cumulative distribution function $G_a(a)$ and density $g_a(a)$. Firms hire workers whose

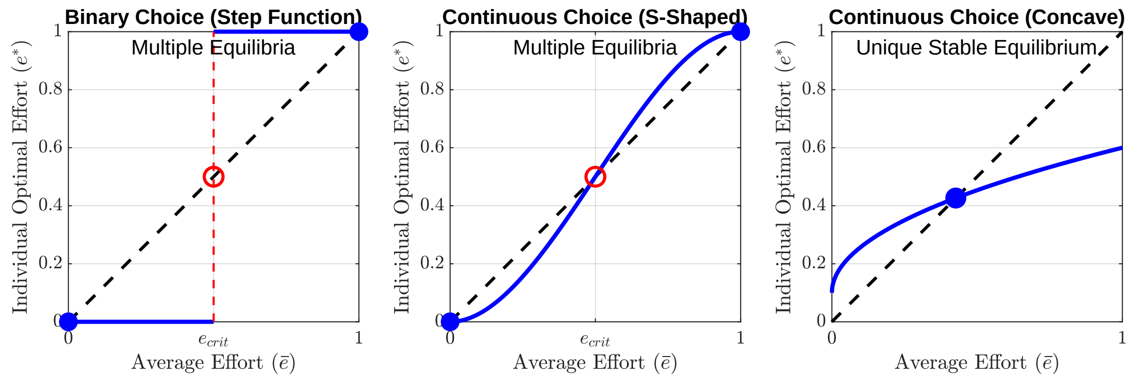

Figure A.1: Best response mappings

ability exceeds a cutoff a_c. The average ability among hired workers is given by the truncated mean

$$\bar{a} = \mathbb{E}[a \mid a \geq a_c] = \mu + \sigma \frac{g_a\left(\frac{a_c-\mu}{\sigma}\right)}{1 - G_a(a_c)},$$

where $g_a(\cdot)$ denotes the standard normal density and $1-G_a(a_c)$ is the probability of exceeding the cutoff.

As the cutoff increases, the truncated mean converges to the cutoff itself:

$$\bar{a} = a_c + \frac{\sigma^2}{a_c} + o\left(\frac{1}{a_c}\right), \qquad a_c \to \infty.$$

Hence, under normal ability, selection raises $\bar{a}$ only marginally (by a vanishing additive term) above the cutoff, reflecting the thin upper tail. Screening incentives respond weakly to increases in dispersion, and coordination effects are limited: increasing a_c yields little additional ability. What matters for the Big Push is how strongly $\bar{a}$ responds to increases in the screening threshold:

$$\frac{d\bar{a}}{da_c} \sim 1 - \frac{\sigma^2}{a_c^2} + o\left(\frac{1}{a_c^2}\right) \to 1.$$

With lognormal ability, the right tail is thinner than Pareto but heavier than normal. The truncated mean above cutoff a_c is

$$\bar{a} = \mathbb{E}[a \mid a \geq a_c] = e^{\mu+\sigma^2/2} \frac{1 - \Phi\left(\frac{\ln a_c - (\mu+\sigma^2)}{\sigma}\right)}{1 - \Phi\left(\frac{\ln a_c - \mu}{\sigma}\right)},$$

where $\Phi(\cdot)$ is the standard normal CDF. Asymptotically,

$$\bar{a} \sim a_c \frac{\ln a_c}{\sigma^2}, \qquad a_c \to \infty.$$

Selection raises $\bar{a}$ by a multiplicative logarithmic factor. The truncated mean grows faster than the cutoff but very slowly:

$$\frac{d\bar{a}}{da_c} \sim \frac{1}{\sigma^2}(\ln a_c + 1).$$

Hence, lognormal tails allow some amplification, but the marginal feedback from increasing a_c remains weak.

For the PLN distribution with density

$$g_a(a) = \frac{k}{a}\,\Phi\left(\frac{\ln a - \mu}{\sigma}\right)\exp\left(-k(\ln a - \mu) + \frac{k^2\sigma^2}{2}\right),$$

the truncated mean above a_c does not admit a closed-form expression:

$$\bar{a} = \frac{\int_{a_c}^{\infty} a g_a(a)\, da}{1 - G_a(a_c)}.$$

However, for sufficiently large a_c such that the Pareto tail dominates

$$\bar{a} \sim \frac{k}{k-1} a_c, \qquad a_c \to \infty,$$

and

$$\frac{d\bar{a}}{da_c} = \frac{k}{k-1} > 1.$$

For cutoffs a_c that are not asymptotically large, the lognormal component of the PLN distribution affects both the magnitude of the truncated mean and its derivative. While the asymptotic Pareto tail determines the slope of the truncated mean for high cutoffs, the absolute value of $\bar{a} = \mathbb{E}[a \mid a \geq a_c]$ and its derivative in economically relevant ranges depend on σ^2, the variance of the lognormal component. Hence, the lognormal component affects the body of the distribution, even though it does not alter the asymptotic tail behavior.

In summary, normal and lognormal distributions attenuate the feedback from coordination, while Pareto-type tails deliver the strong complementarity between effort and screening that potentially enables a Big Push.

Productivity distribution Ulyssea, 2018 argues that a PLN fits well many salient features of firm size distribution. In a PLN distribution, the right tail remains Pareto. Because the "good" equilibrium is driven by the most productive firms—who are the ones that choose to formalize/export, screen workers more intensively, and pay the highest wages—the Pareto tail ensures that the complementarity between firm screening and worker effort remains strong enough to sustain an S-shaped best-response function (see section K). This allows the economy to maintain two stable equilibria.

M Solution method

The model is solved using a nested fixed-point algorithm that combines general equilibrium conditions with firm-level optimization. The outer loop searches for the domestic demand shifter A such that the expected value of entry equals the fixed entry cost. For each candidate A, an inner root-finding step determines the foreign demand shifter $\tilde{A}$ that satisfies the analogous entry condition in the foreign market. Entry values depend on expected profits, which in turn reflect firm productivity and endogenous decisions regarding formality and exporting. Productivity is discretized over a Pareto distribution using a non-uniform grid, with greater density near the lower bound to better capture firm heterogeneity; the grid consists of 2,000 points. Aggregate outcomes are computed by integrating firm-level behavior over this distribution, ensuring consistency with general equilibrium.